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Impact of the Analytical Geometry Teaching Based on Flipped Classroom and Augmented Reality (AR) Assistance on 3D Geometric Thinking Skills

Prahesti Tirta Safitri*, Mohamad Syarif Sumantri and Makmuri
Educational Technology Study Program,
Universitas Negeri Jakarta,
Indonesia

Abstract. This study aims to examine the impact of the analytical geometry teaching based on flipped classroom and augmented reality (AR) on 3D geometric thinking skills. This study used a quasi-experimental method involving 250 students. Participants were divided into two equal groups. The experimental group received the analytical geometry teaching based on flipped classroom and augmented reality (AR), while the control group received traditional geometry teaching interventions. Several instruments were developed in this study, including a 3D thinking ability scale, 3D animation-based teaching materials, and the development of a geometry training system, along with the implementation and assessment of geometry teaching supported by the training system. Data analysis used in the study included regression tests, t-tests, and ANCOVA tests to investigate the impact of the intervention on students' 3D geometric thinking skills. The results showed that analytical geometry teaching based on flipped classroom and augmented reality (AR) effectively improved 3D geometric thinking skills in all six dimensions. The most significant improvement in 3D geometric thinking skills, from the lowest to the highest, was found in the manipulation dimension, nets dimension, structuring dimension, calculation dimension, properties dimension, and finally the comparison dimension. All of these improvements occurred because the web media and AR scaffolding presented interactive 3D animations depicting the nets of 3D object spaces, thereby enhancing students' imagination and improving their understanding of 3D geometric space. This study implies that the integration of flipped classrooms and AR technology in 3D analytical geometry teaching can improve 3D geometric thinking skills.

Keywords: analytical geometry; flipped classroom; augmented reality (AR), 3D animation; 3D geometric thinking skills

*Corresponding author: Prahesti Tirta Safitri; *prahesti_9902922003@mhs.unj.ac.id*

1. Introduction

Currently, traditional mathematics instruction is considered ineffective in learning mathematics, including teaching analytical geometry. Along with technological developments, various technologies have begun to be applied in mathematics teaching, particularly in teaching analytical geometry (Mjenda & Kyaruzi, 2025; Triviño-Tarradas et al., 2022). Teaching using 3D images is considered incapable of facilitating students' understanding of the representation and properties of geometric components.

Some component properties that require realistic image displays include side length, number of sides, correlations between components, and the correlation of angles and edges that form the structure of geometric objects (Frenken et al., 2024; Simsek et al., 2025). For example, the components of prime objects are the same as those of polyhedra. However, in a more detailed explanation, some components that are considered similar are angles, edges, surfaces, and faces. However, these components actually have different characteristics between the two (Fényes et al., 2022; Lee et al., 2023).

Based on previous studies, the majority of students were unable to identify the components that form the same edges in net drawings compared to 3D objects and were unable to understand the properties of analytical geometry components (Abakah & Brijlall, 2024; Mjenda & Kyaruzi, 2025). This is due to geometry teaching methods that are still limited to the use of instructor drawings, lectures, and illustrations in textbooks. The scaffolding employed in teaching analytical geometry mathematics should enhance students' skills in recognizing the characteristics and elements of 3D shapes; the capability to examine 3D objects can aid students in performing comparative analysis.

One scaffolding that can enhance this ability is augmented reality (AR) technology that presents 3D animations (Gerla & Miranda, 2020; Khajehei et al., 2022). Through this technology, students can easily understand the ability to analyze geometric properties and components. However, teaching 3D geometric concepts limited by formulas will make it difficult for students to compare the volumes of two cylinders made from paper (Abdul Hanid et al., 2022; Triviño-Tarradas et al., 2022). This instance highlights the significance of spatial awareness and visualization abilities. The current focus of research in geometry teaching is thinking skills, visualization skills, and the use of technology.

To improve 3D geometric thinking skills, thinking skills and levels of thinking are grouped for appropriate management (Astatke et al., 2025; Turgut, 2022). 3D geometric thinking skills require training that encourages students to use them; students require 3D geometric thinking abilities to visualize, interpret, and depict 3D geometric forms during their learning of 3D geometry. Previous studies have shown that spatial skills can be enhanced through augmented reality (AR) technology, which presents geometric objects in 3D animations (Kus & Newcombe, 2025; Weigand et al., 2024). AR is currently beginning to be used in educational practice to facilitate student interaction with digital materials. The majority of previous studies have focused on investigating the impact of AR-assisted geometry instruction on students' geometry skills, spatial abilities, and motivation (Özkul, 2025; Siller & Ahmad, 2024). The difference between this study and previous studies lies in the AR virtual medium, which allows students to interact with the learning materials. In addition, is the use of a flipped classroom model that provides opportunities for students to access teaching materials online, and exploration of the impact of AR scaffolding on 3D geometric thinking skills. Based on this explanation, the researcher formulated the problem in this study, namely, what is the impact of teaching analytical geometry based on the flipped classroom model and assisted by AR on 3D geometric thinking skills?

2. Literature Review

The theoretical study used in this research is teaching 3D geometry with AR, as well as the use of the flipped classroom model in mathematics teaching. The explanation includes the theory underlying teaching 3D geometry with AR and practical applications. Furthermore, theoretical study is reinforced by the use of the flipped classroom model in mathematics teaching.

2.1 3D geometry teaching and augmented reality

Assessment of 3D thinking skills includes six aspects: the capability to recognize 3D shapes, the capacity to articulate and explain representations of different 3D geometric objects, the skill to build 3D cubes, the ability to examine the characteristics of 3D geometric shapes, the aptitude to compute volumes and areas of 3D geometric figures, and the ability to contrast attributes of 3D geometric shapes (Huang et al., 2023; Lane & Sorby, 2022). AR scaffolds provide virtual objects alongside real environments, helping students visualize intricate spatial relationships and abstract ideas. Learning using AR provides more comprehensive learning opportunities because it enhances students' attention and motivation (Avcu, 2019; Hidajat, 2023).

Furthermore, AR scaffolds can provide different perspectives on difficult objects. AR-based teaching can improve students' spatial abilities, practical application skills, conceptual understanding, and inquiry-based learning (Frenken et al., 2024; Oughton et al., 2024). AR-assisted geometry teaching can enhance spatial abilities, thereby enhancing understanding of analytical geometry. AR scaffolds assist students in accomplishing spatial visualization tasks, including mentally altering, rotating, and flipping objects (Abakah & Brijlall, 2024; Simsek et al., 2025). AR has the potential to improve spatial visualization and mental rotation abilities and can reduce cognitive load by facilitating a natural interaction between visual abilities and rotation skills (Fényes et al., 2022; Mjenda & Kyaruzi, 2025).

AR-based geometry learning integrates real-life scenarios into the learning process, connecting geometry learning to real-life situations. The use of AR can facilitate students' visualization of objects and can create a learning environment that can create a visual representation of knowledge (Miyazaki et al., 2024; Mwadzaangati et al., 2022). Additionally, AR scaffolding can showcase dynamic, interactive, and organic 3D forms, allowing students to engage with the 3D scaffolding using hand movements. Students are encouraged to enhance their geometric shape abilities through hands-on involvement, creating easily memorable expertise (Gerla & Miranda, 2020; Triviño-Tarradas et al., 2022). The engagement of teachers, students, and 3D geometric resources can foster a positive learning atmosphere. Utilizing AR scaffolding enables 3D content to be observed from various angles, providing students the chance to investigate information at their own speed (Abdul Hanid et al., 2022; Wu et al., 2024). In flipped classroom education, AR plays a vital role in enhancing student learning outcomes, motivation, critical thinking abilities, and self-confidence.

2.2 Using the flipped classroom in mathematics teaching

The main idea of the flipped classroom model stems from the flexibility of the learning environment, adapting the learning culture, instructional design, and skilled teachers. This flipped classroom approach restructures the conventional classroom by encouraging active participation and minimizing teacher-centered learning (Boonstra et al., 2025; Lane & Sorby, 2022).

This shift in the central role from the teacher to the student allows the flipped classroom model to optimize active student participation in the learning process. Learning using the flipped classroom model in mathematics positions the teacher as the director of the learning scenario through appropriate resources and provides opportunities for students to learn outside the traditional classroom (Kus & Newcombe, 2025; Villa-Ochoa et al., 2023). Components of the flipped classroom include meaningful assignments, a supportive transformation of the teacher's role, increased interaction based on instruction, a focus on holistic learning and adjustment to scholastic behavior, rapid feedback that improves learning quality, enhanced technology integration, and timely instruction delivery according to learning needs (Pratama & Yelken, 2024; Siller & Ahmad, 2024).

Essentially, the flipped classroom model is a learning model that offers instruction delivered through an online platform and gives students the opportunity to access and learn material at their own pace (Ortiz-Laso et al., 2023; Weigand et al., 2024). Students can study the material before participating in the classroom learning process and they are provided opportunities to create interactive and dynamic learning processes, including discussion activities, student-centered assignments, and collaborative problem-solving. The flipped classroom model is seen as a blended learning model that combines two or more teaching approaches.

3. Methodology

3.1 Design and participants

This study used a quasi-experimental research method to investigate the impact of flipped classroom-based analytical geometry learning and augmented reality on 3D geometric thinking skills. The 3D geometric thinking competencies investigated covered six dimensions: the ability to identify and create 3D geometric objects, the ability to draw and interpret representations of various 3D geometric views, the ability to arrange 3D cubes, the ability to identify the properties of 3D geometric shapes, the ability to calculate the volume and area of 3D geometric objects, and the ability to conduct comparative analysis of 3D geometric features. This study involved a sample of 250 college-level students aged 20-25 from one of the universities in Indonesia. The population of this study was college-level mathematics students in Indonesia. The gender composition of participants was 55% female and 45% male. Participants were divided into two equal groups. The experimental group received a flipped classroom-based analytical geometry teaching intervention with AR support, while the control group received only conventional analytical geometry instruction using images in textbooks and physical geometric shapes. All participants completed a consent form, ensuring voluntary participation.

All research data were anonymous and used solely for research purposes. All participants took an initial pre-test of 3D geometric thinking skills across all dimensions using 50 questions. The pre-test questions were piloted on a sample to verify their validity. Validity testing was conducted using confirmatory factor analysis. Based on the analysis results, the questions met validity criteria and could be used in the study. Based on the pre-test analysis, all participants demonstrated equivalent 3D geometric thinking skills, allowing for intervention. A 3D geometric thinking ability scale was developed to evaluate 3D geometric abilities.

3.2 Research instruments

This study utilized several research instruments, including a 3D geometric thinking ability test, an AR-based geometry training system, and web media as a facility for the flipped classroom model.

3.2.1 3D geometric thinking ability test

3D geometric thinking ability was assessed using the 3D geometric thinking framework by Pittalis, Mousoulides, and Christou, which emphasizes six dimensions: the ability to identify and create 3D geometric objects, the ability to draw and interpret representations of various 3D geometric views, the ability to construct 3D cube structures, the ability to identify the properties of 3D geometric shapes, the ability to calculate the volume and area of 3D geometric objects, and the ability to perform comparative analysis of 3D geometric features.

This instrument was evaluated through empirical testing on 50 learners to determine validity and reliability. The researcher applied Pearson's product-moment correlation analysis to determine validity and employed the split-half method to evaluate its reliability. This method relies on a straightforward evaluation of the extent of linear correlation among the questions. According to the analysis results, the mean Pearson correlation coefficient across all questions was 0.85. The dimension table of 3D geometric thinking ability is presented in Table 1.

Table 1: 3D geometric thinking ability

No	3D geometric thinking skills	Description
1	The ability to identify and create 3D geometric objects.	The capacity to identify and create nets for three-dimensional geometric shapes and to build these shapes when the net is folded.
2	The capability to sketch and understand depictions of different 3D geometric perspectives.	The capacity to create perpendicular or parallel projections of 3D items and transform them into different forms.
3	The ability to construct 3D cube structures.	The capacity to determine the quantity of unit cubes required to transform an object into a prism utilizing unit cubes, ascertain the count of unit cubes necessary to build a 3D prism, and compute the number of unit cubes that can be accommodated within an empty box.
4	The capacity to recognize the characteristics of three-dimensional geometric forms.	The capacity to recognize prisms and pyramids and understand their characteristics and elements.
5	The capability to compute the volume and surface area of three-dimensional geometric shapes.	The skill to determine the volume and surface area of shapes through unit cubes and nets, along with the capability to compare various volumes.
6	The ability to perform comparative analysis of 3D geometric features.	The capacity to assess attributes and components of different 3D forms, including edges, sides, and angles.

3.2.2 Development of an AR-based geometry training system

The geometry training system based on AR employs 42 educational scenarios. The educational resources were created utilizing Unity3D. Moreover, virtual buttons are employed to enhance student engagement with the AR-driven educational resources. These digital buttons facilitate switching between perspectives and accessing the AR-oriented geometry learning resources. Transitions between views, controlled by the user, are available on mobile devices and provide enhanced visualization choices. Moreover, a matrix system is utilized to enhance the performance of the virtual buttons and establish which view the student will engage with. The operational phases of the AR-based geometry training are as follow: initially, students choose the geometric figure they want to explore, such as a triangular prism.

Subsequently, they engage with the geometric shape by utilizing the corresponding virtual controls. Students can guide the prism's movement as needed to examine the prism's edges from various angles. Students can control the prism's motion by using the virtual buttons. All activities for learning geometry are created utilizing a web page. This online platform features a home menu, enabling students to access geometry resources or materials in accordance with their assignments. The validity of this instrument was tested using an empirical test on the same group of participants to measure construct validity. The test results showed reliability that met the criteria with a Cronbach's alpha value of 0.92. Based on this value, the instrument can be used in research. All of these instruments were used in the pre-test and post-test phases.

3.2.3 Web media as a facility for the flipped classroom model

3D geometry learning is also implemented using a flipped classroom model. Students can access 3D geometry materials outside of class, allowing them to deepen their understanding of the material comprehensively before engaging in classroom learning. The web media used is a learning portal provided by the university, connected to an AR geometry scaffold. This web media has several features, including 3D geometry object materials, discussion, assignments, and assignment submission. All of these features are accessible to students outside of class.

3.3 Procedure

Initially, all participants in both groups took a pre-test to assess their initial 3D geometric thinking skills. The pre-test was conducted to ensure that both groups had equivalent 3D geometric thinking skills, with the same type of test being used in both groups. After ensuring all participants had equal abilities, students received intervention training using a flipped classroom-based geometry learning model and AR-assisted learning.

Students were given a series of analytical geometry tasks, including identifying and creating 3D geometric objects, drawing and interpreting representations of various 3D geometric views, constructing 3D cube structures, identifying the properties of 3D geometric shapes, calculating the volume and area of 3D geometric objects, and conducting comparative analyses of 3D geometric features. This series of tasks was completed both in and out of class. Prior to the 3D analytical geometry learning, students were able to learn 3D geometry through web-based media to prepare them for classroom learning. This flipped classroom-based, AR-assisted analytical geometry teaching intervention was implemented in the classroom for one semester, with two weekly sessions over a six-month period.

Each session focused on the competency of 3D geometric thinking skills, encompassing the six dimensions mentioned above. The control group used the same materials and tests as the experimental group. The only difference lies in the absence of web media; learning is conducted solely through conventional, in-person classrooms. After the intervention, all students took a post-test to investigate differences in 3D geometric thinking skills between the groups. The post-test was conducted to determine the effectiveness of the intervention on 3D geometric thinking skills. The test used in the post-test for the experimental group was the same as for the control group.

3.4 Data analysis

The data analysis used in this study included regression tests, t-tests, and ANCOVA tests to investigate the impact of the intervention on students' 3D geometric thinking skills. The regression test was conducted to determine the significance of the intervention's contribution to 3D geometric thinking skills. The t-test was used to investigate the impact of the intervention on 3D geometric thinking skills based on gender across the six dimensions of 3D geometric thinking skills. ANCOVA tests were conducted to investigate the impact of the intervention on the six dimensions of 3D geometric thinking skills during the pre-test and post-test phases.

4. Result

The instructional method for the 3D geometry class was identical for both the experimental and control groups. The sole distinction was the web and AR media framework, utilized exclusively by the experimental group. The control group employed a projector, authentic 3D geometric shapes, books, and images. Both groups underwent testing with the 3D thinking ability scale shown in Table 1. Additionally, Table 2 displays the findings from the analysis of student demographic information concerning their experiences with computers, among other factors, for both groups. The analysis of the regression coefficients from the model focusing on the gender variable showed no notable differences between female and male students (χ^2 = 32.34, p = 0.97), and the outcomes of the model fit evaluation remained unchanged. Hence, the CFA analytical model employed fulfilled the requirements.

Table 2: Demographic profile of participants

N = 250	Category	Frequency	Percentage %
Gender	Male	102	42
	Female	148	48
Taking online math lessons	No	100	40
	Yes	150	60
Experience using computers (in years)	<1	100	40
	1–3	100	40
	>3	50	20

Table 3: Results of model adaptation analysis

Test	Result	Criterion	Decision
χ2/df	1.72 (*p* = 0.070)	<2 (*p* > 0.5)	High compliance
CFI	0.987	>0.10	High compliance
NFI	0.968	>0.10	High compliance
RMSEA	0.042	<0.05	High compliance

Table 4: Results of descriptive analysis and regression coefficients of 3D Geometric Thinking Ability

	Mean	SS	Regression ratio
Properties	4.63	2.83	64
Structuring	5.21	2.81	58
Manipulation	7.21	2.70	56
Nets	5.63	2.45	54
Calculation	4.52	2.63	52
Comparison	3.57	2.72	3.72

The assessment of 3D geometric thinking skills involves six dimensions: nets, manipulation, structuring, properties, calculation, and comparison. The net's dimension refers to a student's skill in recognizing and producing 3D geometric shapes. The manipulation dimension refers to a student's skill in creating and understanding representations from different perspectives of 3D objects. The structuring aspect pertains to the capability of organizing 3D geometric cubes. The properties dimension refers to the capability of organizing 3D cube configurations, while the calculation dimension pertains to the capacity to compute the area and volume of 3D geometric figures. The dimension of comparison refers to the capability to evaluate the attributes of 3D geometric forms. According to the outcomes of the model estimation analysis evaluated by the 3D geometric thinking ability scale, all regression coefficients demonstrated a notable rise as shown in Table 4. Each item employs a scale of 10 to evaluate the degree of success of all measurement items shown in Table 4.

According to the analysis results, the dimension that had the highest average value was manipulation ability (7.21), while the dimension with the lowest average value was comparison ability (3.57). Additionally, the analysis findings indicate that the greatest variance was observed in the property dimension (SS = 2.83), while the lowest variance was noted in the nets dimension (SS = 2.45). Table 5 displays the outcomes of the analysis regarding the difficulty index and discriminatory power of the questions from the 3D geometric thinking ability test.

According to Table 5, it was observed that the questions' difficulty level was categorized as average, and the discriminatory power value exceeded 0.3. Furthermore, the mean item difficulty index for the 50 questions had a P value of 0.64, the mean discriminative index (D) demonstrated a value of 0.78, and the reliability coefficient was 0.832.

Table 5: Results of the analysis of the difficulty index and the discriminating power of the 3D geometric thinking skills test items

	Item no.	*P*	*D*
Ability to recognize and construct nets			
Identifying cuboid nets	28	0.64	0.49
	41	0.55	0.62
Identifying pyramid nets	35	0.42	0.35
	36	0.39	0.42
Identifying cylindrical nets	24	0.37	0.38
	40	0.69	0.50
Identifying cube nets *	11	0.49	0.47
	37	0.70	0.54
	42	0.72	0.51
Parallel surface analysis of an open cube net *	23	0.54	0.49
Conic net analysis *	25	0.69	0.50
Manipulation of 3D geometric shapes Representation mode			
Ability to translate orthogonal to isometric views	47	0.45	0.41
Ability to translate side projection views to orthogonal views	29	0.70	0.58
Understanding parallel sides of a cube in isometric views	8	0.68	0.60
Understanding vertical sides of a cube in isometric views	7	0.48	0.63
Calculating the sides of triangles, rectangles, and triangular pyramids in transparent views	4	0.69	0.46
	16	0.65	0.68

	17	0.67	0.69
	18	0.70	0.62
Calculating the sides of a triangular prism in transparent form	22	0.63	0.68
Assembling a 3D cube arrangement			
Calculating the cubes required to transform an object into a cuboid	5	0.64	0.57
	13	0.60	0.68
Counting cubes and blocks in a box	1	0.66	0.48
	20	0.53	0.56
Counting cubes in a non-empty box	15	0.58	0.74
	33	0.45	0.43
Calculating the number of cubes in an open box	6	0.68	0.45
Calculating the number of cubes that can be filled in a closed prism	3	0.50	0.42
Understanding the properties of 3D geometric shapes			
Understanding cuboids	44	0.60	0.48
Understanding cuboids with a given number of vertices	49	0.63	0.59
Understanding cuboids with a given number of edges	48	0.57	0.75
Understanding cuboids with a given number of sides	50	0.57	0.74
Understanding pyramids	45	0.61	0.57
Understanding the sides of pyramids	28	0.38	0.48
Understanding the vertices of pyramids	26	0.61	0.73
Understanding the sides of pyramids	27	0.48	0.64
Understanding the height of pyramids*	21	0.34	0.42
Calculating the volume and area of geometric shapes			
Calculating the area of geometric shapes formed by cubes	14	0.53	0.60
Calculating the area of geometric shapes in the form of open nets	9	0.54	0.49
Calculating the volume of cuboids in the form of open nets	12	0.42	0.30
	38	0.40	0.54
Comparing the capacities of rectangular and cylindrical reservoirs	39	0.35	0.38
Calculating the volume of a cylinder in open net form*	10	0.60	0.62
Calculating the height of a prism displayed as volume*	19	0.57	0.66
Comparing the properties and shapes of 3D geometry			
True/false statements about the elements and properties of three geometric solids	43	0.48	0.54
	46	0.40	0.33
Investigating Euler's Rule in	35	0.48	0.60
Extension of Euler's Rule for Prisms	51	0.40	0.43
Comparing the lengths of elements in prisms and pyramids*	31	0.35	0.38
	32	0.70	0.53

Next, the results of the binary correlation analysis on the 3D geometric thinking ability subscale are presented in Table 6. Based on the analysis, all dimension correlations were at a moderate and significant level with a p value < 0.01. The property and comparison dimensions exhibited the strongest correlation, registering a value of 0.68. The manipulation and calculation dimensions exhibited the lowest correlation value, which was 0.52. An independent sample t-test was used to assess the connection between participant gender and 3D geometric thinking ability. The findings of the analysis are shown in Table 7. The examination revealed a notable disparity in the perspectives of female and male students. Regarding perspectives, female students excelled with the values ($t = 2.06$, $p < 0.05$). Nonetheless, no notable variations emerged between genders in different facets of cognitive ability.

Table 6: Binary correlation coefficients for 3D geometric thinking ability subdimensions

	Nets	Manipulation	Structuring	Properties	Calculation	Comparison
Nets	1					
Manipulation	0.62**	1				
Structuring	0.58**	0.65**	1			
Properties	0.64**	0.63**	0.62**	1		
Calculation	0.55**	0.52**	0.61**	0.58**	1	
Comparison	0.57**	0.57**	0.56**	0.68**	0.56**	1

** 0.01 significance level

Table 7: Results of the t-test of the average score of 3D geometric thinking ability based on gender

	Means		*t*	*P*
	Female	Male		
Nets	5.73	5.35	1.40	0.21
Manipulation	6.68	5.85	2.06	0.04*
Structuring	5.04	5.31	−1.00	0.35
Properties	4.62	4.62	−0.32	0.78
Calculation	4.52	4.43	0.12	0.92
Comparison	3.70	3.70	0.10	0.94

*0.05 significance level

Additionally, to assess the efficacy of geometry instruction using flipped classrooms and AR support, a study was carried out comparing the mean scores of 3D geometric thinking abilities. The pre-test was administered to evaluate 3D geometric thinking skills prior to the intervention, and the post-test was administered to evaluate 3D geometric thinking skills following the geometry class intervention. An ANCOVA test was performed to analyze the data.

Furthermore, assessments of homogeneity, normality, linearity, variance homogeneity, and regression slope were performed prior to the ANCOVA test. The normality assessment was conducted by analyzing the skewness and kurtosis values of the data. According to the analysis findings, the skewness and kurtosis fell between 2.0 and -2.0 for both the pre-test and post-test scores.

Additionally, the analysis results showed that the data from the pre-test and post-test exhibited linearity. Furthermore, the assumption of equal variance was not breached either, with a result of ($F(1, 105) = 1.24$, $p = 0.34$). The analysis results indicated that the homogeneous regression slope satisfied the criteria, presenting a value of ($F(1, 105) = 7.62$, $p = 0.01$). According to the analysis results, there was no notable difference in the pre-test scores between the two groups, showing a value of [t (105) = -0.87, experimental = 23.41, control = 22.05, p > 0.05]. In the post-test, students in the experimental group demonstrated a marked improvement in 3D geometric thinking skills with a score of (M = 32.63, SD = 8.24), surpassing the control group, which scored (M = 25.21, SD = 8.10). Table 8 displays the findings from the analysis of 3D geometric thinking abilities in both groups. According to the analysis of geometric thinking skills prior to the intervention shown in Table 7, students in both groups exhibited 3D thinking skills at a level scoring (p > 0.05). Additionally, following the intervention, the mean score of students' 3D thinking abilities exhibited a notable rise, particularly in the structuring aspect of 3D cube configurations and the aspect of volume computations. However, in the dimension of calculating the volume and area of geometric shapes, the increase was not statistically significant with a p value of (= 0.047.

Furthermore, in the subfactor analysis of 3D geometric thinking ability, a statistically significant difference was found, indicating that the experimental group's score was higher than the control group at (p<0.05). Based on the results of the analysis, the most significant difference was found in the dimension of the ability to identify and create 3D geometric shapes with a value of (p<0.01). The overall difference on all scales showed that the experimental group's 3D geometric thinking ability was better than the control group with a value of (p<0.05). Thus, it can be concluded that flipped classroom-based analytical geometry teaching and AR assistance can improve students' 3D geometric thinking ability.

Furthermore, to test the correlation between 3D geometric thinking ability and student gender, an independent sample t-test was conducted. Based on the analysis results, a significant difference was only found in the structural dimension, with female students scoring better [t(103) = −2.06 compared to male students, with scores of 5.62 for females and 4.63 for males, respectively, p < 0.05]. Furthermore, no significant differences were found between genders in other 3D geometric thinking skills.

Table 8: Pre-test and post-test results based on subfactors of the 3D geometric thinking skills scale

		Control		Experimental			
3D thinking level	Phase	*M*	SD	*M*	SD	*T*	*P*
Nets	Pre-test	4.92	2.21	5.34	1.87	–0.89	0.492
	Post-test	5.76	2.24	6.88	1.88	–2.84	0.007**
Manipulation	Pre-test	5.34	1.82	5.25	2.45	0.28	0.794
	Post-test	5.42	2.56	7.56	1.89	–2.42	0.020*
Structuring	Pre-test	4.21	2.35	4.52	2.21	–0.72	0.552
	Post-test	4.83	2.40	6.53	2.43	–1.42	0.007
Properties	Pre-test	2.42	1.80	3.72	2.10	–0.96	0.357
	Post-test	2.93	2.24	5.80	2.45	–2.03	0.042*
Calculation	Pre-test	2.87	1.70	3.45	2.20	–1.04	0.324
	Post-test	3.84	1.89	6.40	1.93	–1.82	0.047
Comparison	Pre-test	2.55	1.73	2.52	1.98	0.07	0.962
	Post-test	2.83	1.62	3.72	1.94	–2.75	0.010*

**: 0.01 significance level, *: 0.05 significance level.

5. Discussion

This study aims to investigate the impact of flipped classroom-based analytical geometry learning with augmented reality (AR) on students' 3D geometric thinking skills. The findings indicate that flipped classroom-based analytical geometry learning with augmented reality (AR) significantly improved the experimental group's 3D geometric thinking skills in all dimensions compared to the control group. These findings indicate that flipped classroom-based analytical geometry learning with AR is highly effective in improving students' 3D geometric thinking skills.

Improvements were seen in the dimensions of recognizing and creating 3D geometric shapes and virtual manipulative skills. This improvement occurred because the AR scaffolding presented information from various sources and did so in a realistic environment. Students' attention was focused on the virtual manipulatives and the concepts presented. However, the use of physical materials often distracts students from geometric concepts and focuses on other factors. Therefore, a combination of real-world and virtual scaffolding is needed in geometry teaching.

These findings align with previous studies in STEM teaching, which showed that integrating virtual scaffolds and physical learning media can improve STEM skills more effectively than using either alone (Li et al., 2025; Weigand et al., 2025). The current findings also support previous studies showing that AR-assisted biology learning and physical image media can improve biology comprehension more comprehensively than conventional methods using only textbook images (Bertrand et al., 2024; Ng et al., 2020).

Furthermore, AR scaffolding, which presents interactive 3D animations depicting 3D object spatial nets, can enhance students' imagination and enhance their understanding of geometric shapes. Furthermore, web media, used to facilitate flipped classrooms, also provides students with opportunities to deeply understand the geometric properties of various geometric shapes before class begins. Students also have ample opportunities to experiment with understanding the geometry formation process, the components of geometric structures, and their properties through the provided web media and AR scaffolding.

This process significantly improves 3D geometric thinking skills in the dimension of 3D geometric shape representation manipulation compared to the control group. These findings confirm that flipped classroom-based learning and AR-assisted learning can effectively improve representation manipulation skills (Kaźmierczak et al., 2025; Pawlak-Jakubowska & Terczyńska, 2023). This finding aligns with several previous studies that confirmed that AR-based visual elements can improve 3D geometric thinking skills in the transition dimension between representations (Arik Karamik et al., 2025; Siller & Ahmad, 2024).

The next finding is that flipped classroom-based geometry learning with AR assistance can improve students' mental rotation abilities. This improvement occurs because the AR scaffold is able to present 3D manipulation representations within the AR scaffold. This finding is supported by earlier research indicating that instructing geometry dissertations through virtual manipulation is effective in enhancing reflection and rotation skills, which are sub-skills of 3D geometry (Bergström et al., 2023; Pratama & Yelken, 2024). This discovery is further backed by the notion that students' visualization skills can be enhanced via authentic learning experiences (Frenken et al., 2024; Oughton et al., 2024). Geometry instruction in flipped classrooms, supported by AR, can enhance students' comprehension of 3D geometric shapes by allowing them to observe objects from different angles. The integration of virtual items with real items enables students to link intricate spatial skills with abstract ideas (Bergdahl & Langmann, 2018; Mwadzaangati et al., 2022).

The subsequent discovery indicated that geometry learning through a flipped classroom approach with AR enhanced 3D geometric thinking abilities in terms of building 3D cubes as well as determining the volume and surface area of 3D geometric solids. This finding was reinforced by a previous study showing that some students were able to solve simple 3D prism problems but were unable to solve more difficult problems and complex structures, as they required enhanced visualization skills (Abakah & Brijlall, 2024; Simsek et al., 2025). This finding was further reinforced by a previous study showing that the majority of students who used memorized formulas tended to have difficulty solving problems requiring visualization skills (Mjenda & Kyaruzi, 2025; Sheynikhovich et al., 2025). Most students, when solving volume and space problems, used formulas without considering spatial direction when the problem included formulas.

Another finding was that students in the experimental group showed improved 3D geometric thinking skills in the dimension of determining the properties of 3D geometric shapes. The majority of student errors before the intervention was found in the dimension of determining geometric shapes and properties. This was due to the abstract theorems and concepts presented only in conventional geometry instruction. Flipped classroom-based geometry instruction with AR-assisted learning can bridge the gap between shapes, formulas, and geometric language. The scaffolding of flipped classroom-based and AR-assisted learning materials can facilitate students in viewing geometric objects from various perspectives to investigate the properties of their shapes, such as angles, edges, and so on (Abdul Hanid et al., 2022; Triviño-Tarradas et al., 2022).

This facilitates students in recognizing the differences, similarities, and transformations of 3D geometric shapes (Bozan & Taslidere, 2025; Wu et al., 2024). Furthermore, flipped classroom-based and AR-assisted analytical geometry instruction can also improve 3D geometric thinking skills in the dimension of comparing 3D shape features. Improving 3D comparison skills requires training supported by concrete materials, computers, and 2D representations simultaneously. This is accommodated in the flipped classroom-based and AR-assisted analytical geometry instruction intervention. These findings are supported by a previous studies that demonstrated that the majority of students were able to calculate the volume of a cylinder using information about the base and height (Astatke et al., 2025; Dilek Eryigit et al., 2025). The findings of this study are reinforced by the concept that understanding the hierarchy of geometric concepts requires various prototype forms from various positions (Lane & Sorby, 2022; Turgut, 2022).

6. Conclusion, Implications, and Recommendations

An analytical geometry teaching approach that utilizes a flipped classroom and AR assistance significantly enhanced 3D geometric thinking abilities in all six dimensions. The greatest enhancement in 3D geometric thinking abilities was observed in the manipulation aspect, referring to students' capacity to create and understand representations of different perspectives of 3D objects. This was succeeded by the nets dimension, which refers to the capability to identify and devise 3D geometric forms; the structuring dimension, which relates to the skill of assembling 3D geometric cubes; the calculation dimension, which pertains to the ability to determine the volume and area of 3D geometric figures; the properties dimension, which involves the capability to organize 3D cubes; and lastly, the comparison dimension, which encompasses the ability to contrast attributes of 3D geometric shapes.

This enhancement took place as the AR scaffolding provided information from multiple sources and displayed it in a lifelike setting. Students' attention was focused on the virtual manipulatives and the concepts presented. All of these improvements occurred because the web media facilitated students' online understanding of geometry material before class, thus ensuring adequate understanding. The AR scaffolding also facilitated students' viewing of geometric objects from various perspectives to investigate the properties of their shapes, such as angles, edges, and so on. In addition, AR scaffolding also presents interactive 3D animations depicting 3D object spatial nets, thereby enhancing students' imagination and improving their understanding of geometric solid shapes.

This study implies that the integration of flipped classrooms and AR technology in 3D analytical geometry learning can improve 3D geometric thinking skills. In addition, the integration of software and other technologies in mathematics learning is very important in optimizing student learning outcomes. This study has several limitations, including the study focusing on analytical geometry material, only using a quantitative approach, a sample focused on tertiary level and only one country, the intervention duration is still quite short, and the investigation of the role of flipped classrooms and AR scaffolding on cognitive load and psychological aspects that support mathematics learning is not yet optimal.

Based on these limitations, the researcher recommends several suggestions including the use of web-based media flipped classroom and AR scaffolding needs to be done on other mathematics materials, the need for qualitative analysis to strengthen quantitative findings, the sample needs to be expanded to the secondary school level and from various countries, the duration of the intervention needs to be extended to obtain better results, and further exploration is needed on the role of flipped classroom and AR scaffolding on cognitive load and psychological aspects that support mathematics learning. This research broadly contributes to the use of pedagogical scaffolding in the future, the development of AR-based software in educational practice and curriculum development.

7. References

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