International Journal of Learning, Teaching and Educational Research
Vol. 25, No. 4, pp. 829-864, April 2026
https://doi.org/10.26803/ijlter.25.4.38
Received Jan 5, 2026; Revised Mar 13, 2026; Accepted Apr 14, 2026

Visualizing Mathematics Learning: A Science Mapping of Augmented Reality and Immersive Learning Technologies in Mathematics Education

Ilene DS. Bunag*, Angelita R. Iglesia, Ronilo P. Antonio and Louis Robert C. Sison
Bulacan State University – College of Professional Teacher Education
City of Malolos, Bulacan, Philippines

Abstract. Recent developments in augmented reality (AR), virtual reality (VR), and immersion visualization technologies have created increased interest in their potential to mitigate ongoing challenges faced by mathematics education, especially those related to abstraction, visualization and learner motivation. Despite the growing number of research studies, there remains a lack of understanding of how immersive technologies have shaped mathematics education. This study provides a broad bibliometric analysis of Scopus-indexed (n = 408) publications for 2002–2025 examining augmented and immersive learning technologies in mathematics education. Using citation analysis, document co-citation analysis, co-word analysis, trend topic analysis and three-field plotting via VOSviewer and Biblioshiny, the study charts the field's intellectual landscape, thematic evolution and emerging research frontiers. Recent literature published across almost 3 years indicated a rapidly expanding area of research, with contributions being built on earlier work on basic frameworks for AR and learning applications, predominantly from mathematics. The collective citation and co-word analysis identified four prominent thematic clusters, namely (1) augmented reality applications and mathematics learning processes, (2) STEM integration and technology-enhanced science and mathematics education, (3) immersive and intelligent learning environments, and (4) virtual reality, e-learning, and educational foundations. Trend analyses, in addition, also show a progressive transition from early visualization and computer-aided instructions to more immersive, learner-centered and interdisciplinary STEM research. The study provides a novel mapping of the evolution in bibliometric terms, analogous to the spread of technological novelty to pedagogical efficacy. These outcomes point to mathematics education policies that integrate and implement immersive tools for geometry and

Citation:
Bunag, I. D. S., Iglesia, A. R., Antonio, R. P., & Sison, L. R. C. (2026). Visualizing Mathematics Learning: A Science Mapping of Augmented Reality and Immersive Learning Technologies in Mathematics Education. *International Journal of Learning, Teaching and Educational Research*, *25*(4), 829–864. https://doi.org/10.26803/ijlter.25.4.38

*Corresponding author: Louis Robert C. Sison; *louisrobert.sison@bulsu.edu.ph*

lifelong curricula, and scalable implementations or necessary cognitive scaffolds.

Keywords: augmented reality; virtual reality; immersive learning; STEM education; bibliometric analysis; visualization technologies; Scopus; thematic evolution; science mapping

1. Introduction

Recent digital and immersive technologies are quickly changing how mathematics is taught. These tools help tackle ongoing challenges with abstraction, visualization, and student motivation. Teaching mathematics as an abstract discipline is considered to be conceptually demanding, which has been found to have a negative impact on conceptual understanding and motivation when this approach is used in classroom instruction (Arcavi, 2003; Stylianou, 2010). In reaction to this, researchers and educators have looked more toward technology-enabled learning environments that could externalize abstract mathematical concepts with the purpose of facilitating meaningful learning.

Emphasized among them are related visualization technologies including augmented reality (AR), virtual reality (VR), and other immersive and interactive forms of visualizations. Such technologies allow learners to relate interactive visualizations in three dimensions, physical or abstract representations of mathematical objects and phenomena, and situated and experiential explorations of mathematical concepts in ways that connect between abstract symbols and concrete understanding (Radianti et al., 2020; Makransky & Petersen, 2021). Informed by theories of constructivist and embodied cognition, immersive learning environments are hypothesized to facilitate deep concept learning using the potentials of directing learners to actively generate knowledge through touch, perception and exploration (Dede, 2014; Johnson-Glenberg et al., 2014).

In mathematics education, virtual reality has been used in various topics such as geometry (Limniou et al., 2015; Mora et al. 2017), spatial reasoning, calculus and algebra across educational settings from primary up to higher education. Empirical studies measure positive impact on students' conceptual understanding, spatial ability, motivation and engagement when educational VR tools are synergized with the curriculum rather than as a stand-alone novelty toy (Akçayır & Akçayır, 2017; Ibáñez & Delgado-Kloos, 2018). Nevertheless, concerns persist about the level of instructional design quality, cognitive load measures used, scalability and uneven coverage across different mathematical domains and learning contexts (Parong & Mayer, 2018; Makransky & Petersen, 2021).

However, the literature about immersive technologies in mathematics education has increased rapidly and heterogeneously through empirical and review-based studies across multiple research fields, publication channels and methodological traditions. As the overlap between economic, social, and ethical factors grows more intricate, it is much harder to gain a coherent picture of the field as such, identify more dominant threads of research endeavor, get a sense of where some signs of life are coming from and insight into where further 'stock-taking' should best be done. While several systematic reviews and meta-analyses investigated the efficacy of using AR and VR for education in general (Wu et al., 2013; Radianti et al., 2020), far fewer have categorized knowledge structure, thematic evolution, and conceptual networks of immersive technologies within the context of mathematics education.

Traditional systematic reviews are highly effective at synthesising particular types of empirical evidence around pedagogical effectiveness but can be less adept at mapping so broad intellectual territory or identifying structural relations across so wide a (and fragmented) literature. Hence, bibliometric analysis is an important complementary approach toward filling this gap. Bibliometric functions that can be derived from citational or co-citation and co-word analysis of logically associated words signalling subject-matter (Aria & Cuccurullo 2017) produce insights into influential records which particularly serve to establish formations amongst research clusters and thematic evolution in a field (Donthu et al. 2021). Compared to traditional narrative reviews, bibliometric methodologies provide an objective and data-driven view of literature dynamics, enabling the identification not only of intellectual bases, but also of current and emerging research fronts that occupy academic discourse.

The present research is an extensive bibliometric investigation of articles related to use of immersive technologies in mathematics education via augmented and virtual reality, immersive learning as well as interactive visualization. Utilizing bibliometric techniques—including the VOSviewer and Biblioshiny tools—the researchers aim to (1) identify publication growth trends and significant sources, (2) reveal the intellectual structure of literature through document co-citation analysis, (3) discern dominant, as well as emerging research themes based on co-word topic analyses, and (4) map out temporal progressions of core concepts.

Through consolidating these observations, this paper intends to provide a comprehensive overview of how immersive technologies have impacted mathematics education research for researchers, educators and policymakers alike whilst identifying under-explored domains and future research opportunities. This study is unique in that it offers a granular look at the mathematics-specific literature on immersive technologies, mapping the recent confluence of pedagogical theory with advances in new technology not observed in some STEM-wide reviews.

2. Literature Review

2.1 Immersive Technologies in Education

The use of immersive technologies in educational research, and more specifically AR, virtual reality (VR) and interactive visualization has been increasing over the last couple of decades. The set of computational technologies that bridge between the learners and virtual objects/environments are known as Augmented Reality (AR) technology, which is defined in general as systems designed to increase or duplicate reality by permitting users to interact with digital objects and environments as if they were real time (Azuma, 1997; Radianti et al., 2020). Emerging from constructivist learning theory, immersive technologies facilitate active acquisition of knowledge by embedding it within interactive and meaningful contexts (Dede, 2014). In the context of geometry, immersive technologies operationalize constructivism by transforming abstract symbolic notations into manipulatable 3D objects. From a cognitive standpoint, such environments can be also connected to embodied cognition which claims that learning is embedded into bodily interaction, perception and engagement in space (Abrahamson, 2016; Johnson-Glenberg et al., 2014).

Most systematic reviews in STEM education state that immersive technologies can increase students' motivation, engagement and deepen conceptual understanding when it is pedagogically linked to the learning goals (Akçayır & Akçayır, 2017; Garzón & Acevedo, 2019). Yet, such benefits are not assured; poorly designed immersive environments can add to cognitive workload and interfere with learning goals (Parong & Mayer, 2018; Makransky & Petersen, 2021). Immersive environments are information-rich and often visually complex, which can impose a high "extraneous cognitive load" on the learners. Accordingly, recent studies highlight instructional designs, scaffolding, and its relationship to theories of learning when implementing immersive mediums in education.

2.2 Augmented and Virtual Reality in Mathematics Education

Mathematics education is especially suitable for the use of immersive technology, drawing on its abstract and symbolic nature. Visualization has been acknowledged as an essential aspect of mathematical thinking, especially in the areas of geometry, spatial reasoning and functions (Arcavi, 2003; Stylianou, 2010). AR and VR tools can be used to make abstract representations external, as well as to manipulate dynamically mathematical objects and relationships that are hard to capture in static diagrams or symbolic notation only (Ibáñez & Delgado-Kloos, 2018).

Mathematics education research has also shown that instructions supported with AR can be successful at increasing students' conceptual understanding, spatial abilities, and learning motivation especially in geometry and three-dimensional visualization tasks (Bujak et al., 2013; Arıcan & Özçakır, 2020). Likewise, VR environments have been used to support experiential learning through immersing learners in virtual learning scenarios with a strong mathematical focus (e.g., Kaufmann & Schmalstieg, 2003; Makransky & Petersen, 2021) to encourage exploratory and inquiry-based approaches. Interactive visualization tools, such as 3D learning environments and dynamic geometry software also enrich students' opportunities to make connections between different representations and engage in more sophisticated mathematical reasoning (Sinclair et al., 2016).

Nevertheless, the literature points out unbalanced adherence across mathematical contents and educational levels. Most of the research focuses on geometry and spatial visualization and to a less extent in algebra, calculus, statistics or mathematical proof (Ibáñez & Delgado-Kloos, 2018; Radianti et al., 2020). Many studies are also based on short-term interventions, and little is known about longer term learning impacts or classroom scalability.

2.3 Intellectual and Thematic Developments in the Field

Besides single studies, several other authors have recently tried to draw together evidence on immersive technologies in education from a broader perspective by means of reviews and meta-analyses. These studies describe the emergence of themes including learning outcomes, learner engagement, cognitive load and usability with a growing interest in AI-supported immersive environments attributed for adaptive learning systems (Garzón & Acevedo., 2019; Makransky & Petersen, 2021; Cevikbas et al., 2024). However, these syntheses present notable limitations when applied specifically to mathematics. Commonly, mathematics education is frequently subsumed under general STEM frameworks, blurring the distinct cognitive mechanisms. Moreover, these reviews focus on efficacy rather than the conceptual structure in the field, which leaves a gap in understanding how pedagogical theories evolved alongside technological tools.

Previous bibliometric studies, referring to educational technology also showed that science-mapping techniques (e.g., citation analysis, document co-citation analysis or co-word analysis) proved to be useful for identifying intellectual structure and thematic evolution of the research domains (Aria & Cuccurullo, 2017; Donthu et al., 2021). Although it is true that some bibliometric studies have been carried out, for example, on mobile learning or general AR/VR in education (Wu et al., 2013; Radianti et al., 2020), the researchers have found no specific and detailed bibliometric mapping of immersive technologies in mathematics education. Therefore, foundational theories, dominant research streams, and emerging topics in this domain are explored minimally.

2.4 Research Gap

A synthesis of the literature reviewed revealed the following gaps:

- Fragmented Disciplinary Landscape: While the research output on AR and VR in mathematics education is already significant, its fragmentary distribution across disciplines and publication venues makes it difficult to recognize focused research lines with corresponding intellectual basis.
- Limited Structural Analysis: Previous reviews have focused mostly on effectiveness and learning results with little focus on the conceptual structure, thematic clustering and trend of evolution within this research area.
- Methodological Gap: There is no data-driven, overall mapping that could well combine citation-based influence together with keyword-based thematic analysis is available for uncovering both established and novel research fronts in immersive mathematics education.
- Lack of Evolutionary Perspective: Little bibliometric research has addressed the evolution of research foci brought about by different technological advances (i.e. from early-time visualization aids to recent immersive and AI-augmented environments, along with interactive learning environments) in mathematics education.

Because these specific gaps exist, this study was designed to solve them by answering the research questions on the next section.

2.5 Research Questions

To address these gaps, the present study is guided by the following research questions:

1. What are the publication trends, influential sources, and highly cited documents shaping research on immersive technologies in mathematics education?
2. What is the intellectual structure of the field as revealed through document co-citation networks, and which theoretical and methodological traditions underpin current research?
3. What major thematic clusters and research foci emerge from co-word analysis of author keywords in immersive mathematics education research?
4. How have research topics and thematic priorities evolved over time, and what emerging trends characterize recent scholarship in this field?

By providing an evidence-based overview of the field, the study aims to enable a more strategic and unified vision of immersive technologies in mathematics education and inform future research, design activities, as well as craft policy directions.

3. Methodology

3.1 Bibliometric Approach

Using a bibliometric research design, the researchers conducted a systematic analysis of how the intellectual structure development of augmented reality (AR) and immersive learning technologies for mathematics education has evolved. Bibliometrics analysis allows for the large-scale quantitative exploration of scientific literature by analyzing publication metadata, citation relationship time series, and keyword co-occurrence patterns, which is particularly useful for identifying influential works, dominant research foci, and emerging topics in a field (Zupic & Čater, 2015; Donthu et al., 2021). Incorporating bibliometric indicators (e.g., publication quantity and impact), as well as science mapping methods (co-citation, co-word map, and trend analysis), allows for providing descriptive and relational knowledge on how AR-related research has evolved over time in mathematics education (Aria & Cuccurullo, 2017).

3.2 Search Strategy and String

The literature search was conducted on the Scopus database, as it encompasses peer-reviewed journals, conference proceedings, and book chapters related to education, engineering, and technology. Scopus is commonly used in bibliometric research due to its consistent citation indexing, metadata, and good compatibility with bibliometric analysis tools such as VOSviewer and Biblioshiny (Mongeon & Paul-Hus, 2016).

To maintain the balance between conceptual focus and broad coverage, the search strategy combined two primary concept groups: (1) immersive and visualization-based technologies, on one hand, and (2) contexts related to mathematics education, on the other. Boolean operators (OR, AND) were adopted to cover variations of terminologies within each concept as well as to ensure a distinct overlap between technology and mathematics education areas.
The final search string applied in Scopus was:

("augmented reality" OR "virtual reality" OR "immersive learning"
OR "interactive visualization" OR "3D learning environment")
AND
("mathematics education" OR "math education" OR "mathematics teaching"
OR "mathematics learning" OR "math classroom" OR "mathematics pedagogy")

Table 1: Search String in Scopus Database

No.	Keywords / Operators	Justification
1	**"Augmented reality", "virtual reality", "immersive learning", "interactive visualization", "3D learning environment"**	Captures a broad range of immersive and visualization-based technologies used in educational contexts, ensuring inclusion of both AR- and VR-focused studies as well as related immersive learning environments.
2	**"Mathematics education", "math education", "mathematics teaching", "mathematics learning", "math classroom", "mathematics pedagogy"**	Encompasses diverse instructional, curricular, and pedagogical practices in mathematics education, integrating studies that address multiple grade levels and teaching strategies.
3	**OR (within concept groups)**	Supports the retrieval of studies that use diverse terminology to describe comparable concepts, maximizing recall and reducing the risk of missing relevant literature.
4	**AND (between concept groups)**	Maintains conceptual relevance by limiting results to studies that directly link immersive technologies with mathematics education.

This framework of systematic search strategy was outlined by literature concerning immersive technologies and mathematics education, providing a solid and transparent dataset for subsequent bibliometric analysis.

3.4 Inclusion and Exclusion Criteria

No restrictions were applied in terms of publication year or language, as to ensure that a wide variety of studies including both original and more recent studies were included. These are articles published in peer-reviewed journals, review articles, conference papers, and book chapters probing explicitly how different immersive technologies (i.e., augmented reality, immersive learning, or interactive visualization) are integrated within the teaching or learning of mathematics. To ensure consistency, quality, and bibliographical completeness of the dataset, only articles indexed in the Scopus database were considered.

3.5 Data Cleaning and Preparation

The search results from Scopus were exported to CSV and BibTeX formats to be used with different bibliometric and visualization software. Before the analysis, the data were cleaned and preprocessed to ensure maximum reliability and reduce errors. This included removing duplicates, standardizing author names, journal titles, and keywords, and making sure synonyms were consistent. (e.g., "math education" vs. "mathematics education") to keep the dataset's integrity at the concept level. Furthermore, to ensure the interpretability of our thematic networks, generic / non-informative keywords (e.g., education, study) were excluded. Keyword harmonization was especially crucial for the co-word analysis, as a lack of consistency in terms can fragment networks and misrepresent the nature of conceptual relationships within the field (Callon et al., 1983; van Eck & Waltman, 2014).

3.6 Analytical Techniques

Various bibliometric methods were used to generate a multi-perspective and integrated picture of the AR/IL literature in mathematics education. First, a citation analysis was performed to identify the most influential publications and foundational documents in this domain, as highly cited papers are likely to be seminal contributions that have driven subsequent research and theoretical elaboration (Bornmann & Leydesdorff, 2014). To explore the domain's intellectual structure, VOSviewer was used to analyze document co-citations (i.e., co-occurrences of documents in a reference list) in terms of how often two publications are cited together in the entire dataset. The method helped identify the main research clusters and key theoretical traditions that inform AR studies in mathematics education (Small, 1973; Surwase et al., 2012).

Furthermore, co-word analysis was used in the author keyword to show thematic structure and research hotspots in this field. By exploring the correlations among keywords, this method helped find out the core knowledge structure and key research topics that have influenced AR-assisted mathematics learning (Callon et al., 1983; Biranvand et al., 2020). A trend analysis is used to trace the evolution of this literature over time, and Biblioshiny, an R-based bibliometrics interface, was used to interpret the evolution and growth of main themes. Specifically, this analysis enabled us to identify shifting research foci and emerging frontiers of study in the field (Aria & Cuccurullo, 2017). Third, by using VOSviewer network, overlay and density visualizations were generated that allow to visualize the relationship between documents and keywords focusing simultaneously on structural and temporal aspects of research landscape (van Eck & Waltman, 2010).

4. Results And Findings

4.1 Descriptive Bibliometric Trends and Publication Patterns

The bibliometric dataset included 408 documents published between 2002 and 2025 in 149 sources. This shows a growing body of literature on augmented reality and immersive learning technologies in mathematics education. The field has grown at an annual rate of 15.34%, and the average document is 3.99 years, reflecting the recent and rapid development. The publications accumulated an average of 11.59 citations per document and cited 3,063 references in total. Content analysis identified 1,517 Keywords Plus and 869 Author Keywords, highlighting a wide range of topics. There were 1,130 authors, and most research was collaborative, with only 37 single-authored documents, an average of 3.27 co-authors per paper, and 15.44% of papers involving international collaboration. In terms of publication formats, conference papers (n = 189) and journal articles (n =

150) dominated the corpus, followed by conference reviews (n = 27), book chapters (n = 25), and review articles (n = 12).

Table 2: Main Information about the Bibliometric Dataset

Description	Results
MAIN INFORMATION ABOUT THE DATASET	
Timespan	2002–2025
Sources (journals, books, conference proceedings, etc.)	149
Documents	408
Annual growth rate (%)	15.34
Average document age (years)	3.99
Average citations per document	11.59
Total references	3,063
DOCUMENT CONTENT	
Keywords Plus (ID)	1,517
Author keywords (DE)	869
AUTHORS	
Total authors	1,130
Authors of single-authored documents	35
AUTHOR COLLABORATION	
Single-authored documents	37
Average co-authors per document	3.27
International co-authorships (%)	15.44
DOCUMENT TYPES	
Articles	150
Books	2
Book chapters	25
Conference papers	189
Conference reviews	27
Editorials	1
Errata	1
Retracted documents	1
Reviews	12

4.2 Annual Publication Trends

Research on augmented reality and immersive learning technologies in mathematics education has grown steadily from 2002 to 2025. From 2002 to 2014, only a few studies were published each year, usually fewer than five. Starting in 2015, the number of publications began to rise, with a sharper increase after 2017. The number of studies increased from 16 in 2017 to 26 in 2019 and continued to grow, even with some small changes in 2020 and 2021. The biggest jump happened recently, with 38 publications in 2022, 48 in 2023, and a peak of 78 in 2024 and 80 in 2025. This trend shows that interest in immersive and visualization-based technologies in math education has grown quickly, especially after 2017.

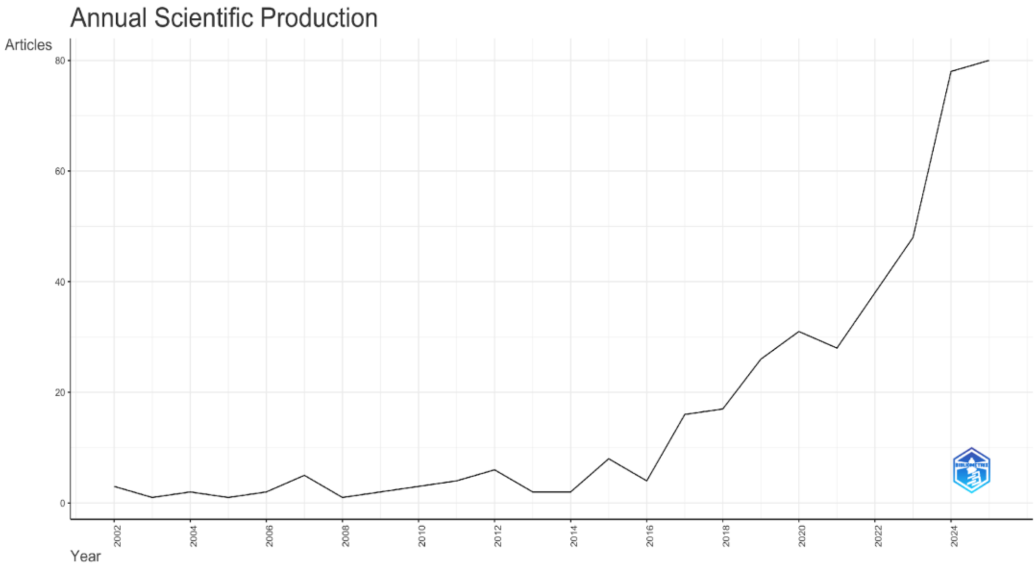

Figure 1: Annual Scientific Production on Augmented Reality and Immersive Learning Technologies in Mathematics Education (2002–2025)

4.3 Geographic Distribution of Research Output

Based on the distribution of publications context and numbers, research regarding augmented reality and immersive learning technologies in mathematics education is being carried out globally. Indonesia topped the list with 127 publications, followed by the United States with 103, and China with 66. Furthermore, other countries producing considerable investigations were Malaysia (n = 36), Turkey (n = 34), Greece (n = 27) and Mexico (n = 26) demonstrating the interest from both developed and developing nations.

In Europe, countries like Italy (n = 22), Germany and Spain (n = 20 each), Portugal (n = 14) and Austria (n = 17) contributed uniformly. Contributions also came from the Asia-Pacific, Latin America and parts of Africa, indicating the global reach of this research. While a great number of countries accounted for fewer publications, the overall distribution presents globally distributed research in an unequal way where selected areas have been more concentrated in doing research.

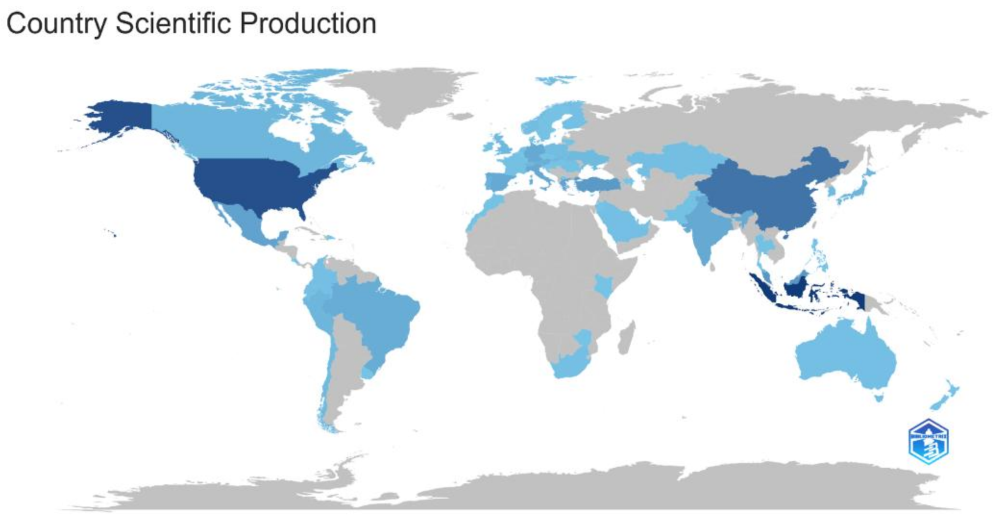

Figure 2: Global Distribution of Scientific Production on Augmented Reality and Immersive Learning Technologies in Mathematics Education

4.4 Most Relevant Sources

The source productivity of the dataset suggests that conference proceedings are the major publication type for sharing research related to augmented reality and immersive learning technologies in mathematics education. The leading source was Journal of Physics: Conference Series, followed by AIP Conference Proceedings and Communications in Computer and Information Science. In terms of top sources, the most robust journal-based contributions were made in Lecture Notes in Computer Science, Lecture Notes in Networks and Systems and Education and Information Technologies. Most research is shared at conferences, while some gets published in high-impact education and computing journals.

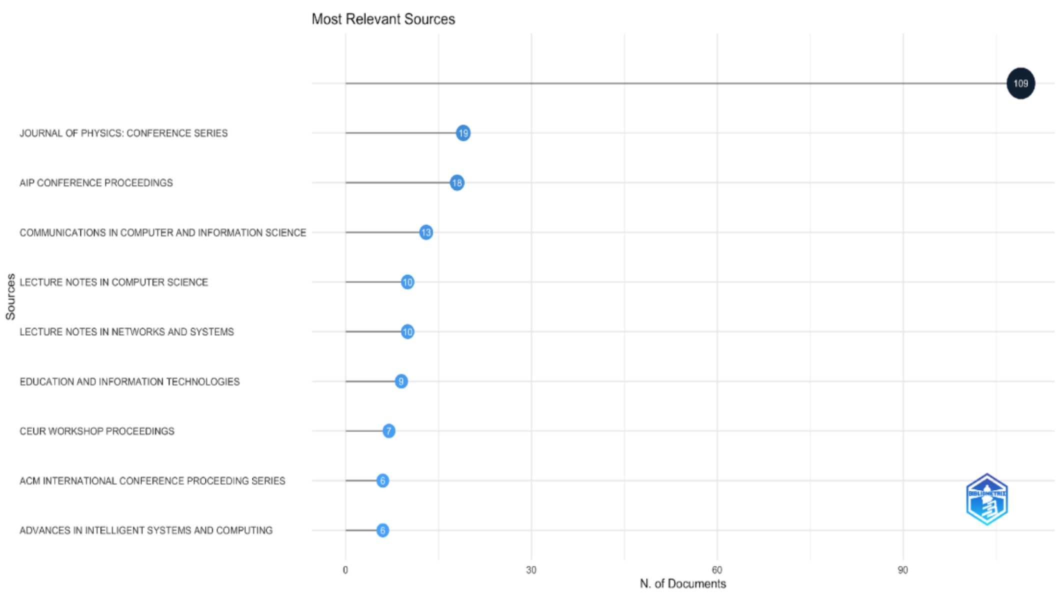

Figure 3: Most Relevant Sources Publishing Research on Teacher Education in Environmental and Sustainability Education (1973–2025)

4.5 Most Prolific and Influential Authors

The author productivity analysis indicates that the augmented reality and immersive learning technologies in mathematics education have few highly prolific authors. Very few authors produced many publications, with between four and six papers from individual authors but a lower contribution for most other authors. This pattern suggests a small core of leading researchers constitutes the heart of the field, over which a broader circle of contributory scholars surrounds with fewer publications. It is a routine bibliometric pattern in studies communities for which excessive authors are essential to the complete establishment and its improvement.

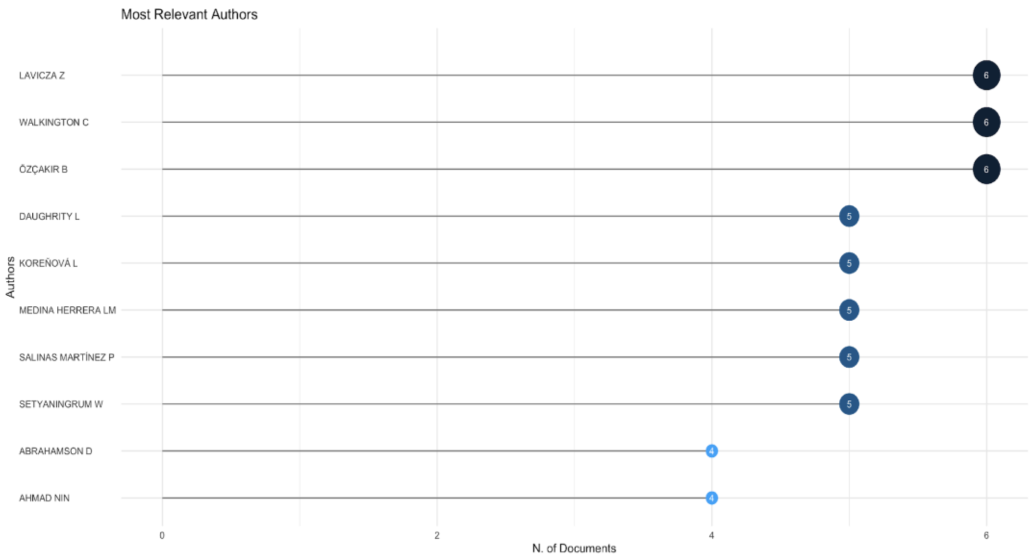

Figure 4: Most Prolific and Influential Authors in Augmented Reality and Immersive Learning Research in Mathematics Education

4.6 Most Productive and Influential Institutional Affiliations

The previous research in mathematics education based on augmented reality and immersive learning technologies is mostly conducted by several highly productive institutions. Universitas Pendidikan Indonesia was the most productive affiliation, with Universitas Negeri Yogyakarta and Beijing Normal University as productive institutions. Their pool of research is from different regions. However, most of their contributions come from certain institutions, as evidenced by the universities in Asia, Europe and North America. The overall distribution indicates that few institutions act as the main center on the field's research output and influence.

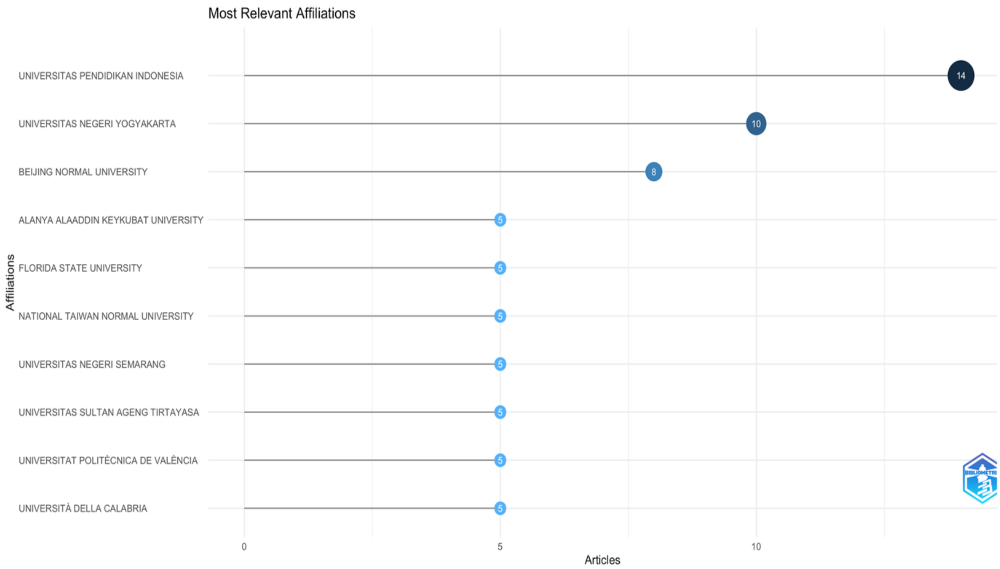

Figure 5: Most Productive and Influential Institutional Affiliations in Augmented Reality and Immersive Learning Research in Mathematics Education

4.7 Citation Analysis

Table 3 presents the most influential publications within the context of augmented reality and immersive learning technologies in mathematics education through acitation analysis. The analysis found that the most cited work in Computers & Graphics was Kaufmann and Schmalstieg (2003), which has had 392 citations. It was the most highly mentioned, clarifying the crucial significance in the beginning stages of augmented reality applications as applied in education. This was in turn followed by Chen's (2019) article in the Journal of Educational Computing Research with 159 citations, followed by Dimitriadou's (2019) article in Education and Information Technologies with 156 citations. It evidences continued interest both for empirical and review contributions.

Table 3: Top 10 Most Cited Documents

Rank	Author(s)	Year	Title (Journal/Source)	Total Citations
1	Kaufmann, H. & Schmalstieg, D.	2003	*Mathematics and Geometry with Collaborative Augmented Reality*	**392**
2	Chen, Y.-C.	2019	*Effect of Augmented Reality on Learning Performance* (Journal of Educational Computing Research)	**159**
3	Dimitriadou, E.	2020	*Augmented Reality in Education: A Systematic Review* (Education and Information Technologies)	**156**
4	Cai, S.	2019	*Augmented Reality-Based Learning in Mathematics* (British Journal of Educational Technology)	**121**
5	Kaufmann, H. et al.	2000	*Construct3D: An Augmented Reality Learning Environment* (ACM SIGGRAPH Conference Proceedings)	**120**

Rank	Author(s)	Year	Title (Journal/Source)	Total Citations
6	Cai, S.	2020	*Interactive Augmented Reality for Mathematics Learning* (Interactive Learning Environments)	**103**
7	Lai, J. W. & Cheong K. H.	2022	*Immersive Learning Using Augmented Reality* (IEEE Access)	**91**
8				
9	Iqbal, M. Z.	2022	*Multimodal Augmented Reality Learning Environments* (Multimodal Technologies and Interaction)	**86**
10	Cascales-Martínez, A.	2017	*Augmented Reality and Mathematics Education* (EURASIA Journal of Mathematics, Science and Technology Education)	**84**

Several highly cited studies on mathematics learning emphasize that specific necessities exist for development of AR applications in dedicated subjects including the field of mathematics. These are Cai (2019) in the *British Journal of Educational Technology* with 121 citations, and also Cai (2020) in *Interactive Learning Environments* with 103 citations. One of the earlier specialisms that remained strongly influential was Kaufmann's (2002) *Construct3D framework*, which now has 120 citations. Also, more recent works by Chen (2020), Lai & Cheong (2022) and Iqbal (2022) have many citations suggesting an emerging impact. The overall citation patterns point to both high impact as well as so-called foundational publications that together make up the intellectual core of the field.

4.8 Co-citation Analysis

Co-citation analysis is a strong bibliometric tool used to reveal the intellectual structure and developmental path of a research field by examining how frequently pairs of documents are cited together in later publications (Surwase et al., 2012). This study used VOSviewer to perform a co-citation analysis and map the key literature that supports research on augmented reality and immersive learning technologies in mathematics education. Out of 2,885 cited references that were taken from 408 documents, 46 references had at least four citations. These often-cited works represent the field's core intellectual base, which includes important ideas and teaching methods that have influenced academic discourse. Table 4 presents the top 10 most co-cited documents, according to their total link strength (TLS), which shows how frequently these works are cited together throughout the body of work.

Table 4: Top 10 Most Co-Cited Documents

Rank	Author(s)	Year	Title (Journal)	Citations	Total Link Strength
1	Azuma, R. T.	**1997**	*A Survey of Augmented Reality* (*Presence: Teleoperators and Virtual Environments*)	36	77
2	Bujak, K. R., Radu, I., Catrambone, R., MacIntyre, B., Zheng, R., & Golubski, G.	**2013**	*A Psychological Perspective on Augmented Reality in the Mathematics Classroom* (*Computers & Education*)	25	52
3	Akçayır, M., & Akçayır, G.	**2017**	*Advantages and Challenges Associated with Augmented Reality for Education: A Systematic Review* (*Educational Research Review*)	21	50
4	Ahmad, N. I. N., & Junaini, S.	**2020**	*Augmented Reality for Learning Mathematics: A Systematic Literature Review* (*International Journal of Emerging Technologies in Learning*)	19	43
5	Azuma, R. T., Baillot, Y., Behringer, R., Feiner, S., Julier, S., & MacIntyre, B.	**2001**	*Recent Advances in Augmented Reality* (*IEEE Computer Graphics and Applications*)	10	28
6	Bacca, J., Baldiris, S., Fabregat, R., & Graf, S.	**2014**	*Augmented Reality Trends in Education: A Systematic Review of Research and Applications* (*Educational Technology & Society*)	7	22
7	Ahmad, F. B. A. R. O.	**2021**	*The Effect of Augmented Reality in Improving Visual Thinking in Mathematics of 10th-Grade Students* (*International Journal of Advanced Computer Science and Applications*)	4	18
8	Arıcan, M., & Özçakır, B.	**2020**	*Facilitating the Development of Preservice Teachers' Proportional Reasoning Using Augmented Reality Activities* (*Education and Information Technologies*)	5	18

Rank	Author(s)	Year	Title (Journal)	Citations	Total Link Strength
9	Chen, Y.-C.	**2019**	*Effect of Mobile Augmented Reality on Learning Performance, Motivation, and Math Anxiety* (*Journal of Educational Computing Research*)	6	18
10	Cai, S., Liu, E., Shen, Y., Liu, C., & Li, S.	**2020**	*Probability Learning in Mathematics Using Augmented Reality: Impact on Students' Learning Gains and Attitudes* (*Interactive Learning Environments*)	4	16

Source: Author interpretation based on VOSviewer analysis

The research on mathematics in augmented reality and immersive learning technologies is located from a set of greatly important and interrelated papers based on co-citation analysis (Table 4). The most co-cited work was Azuma (1997), *A Survey of Augmented Reality*, that presented the highest total link strength (TLS = 77) along with a citation count of 36, indicating the importance as a foundational reference in the field. Next came Bujak et al. (2013) (TLS = 52), Akçayır and Akçayır (2017) or TLS = 50 contributed similarly to improved adoption and understanding of AR in education.

Moreover, some synthesis and empirical studies exhibited notable co-citation collaborations, which included Ahmad and Junaini (2020) (TLS = 43), Chen (2019) (TLS = 18), and Cai et al. (2020) (TLS = 16), reflect the integration of augmented reality frameworks in mathematics learning. However, prior technologies and concepts including those of Azuma et al. (2001) and Bacca et al. (2014) and further solidified the field's theoretical underpinnings. In conclusion, the co-citation structure indicates an aggregate, composite set of studies which underpin contemporary research practice by showing a convergence of systematic educational reviews, mathematics-relevant applications and fundamental augmented reality theory.

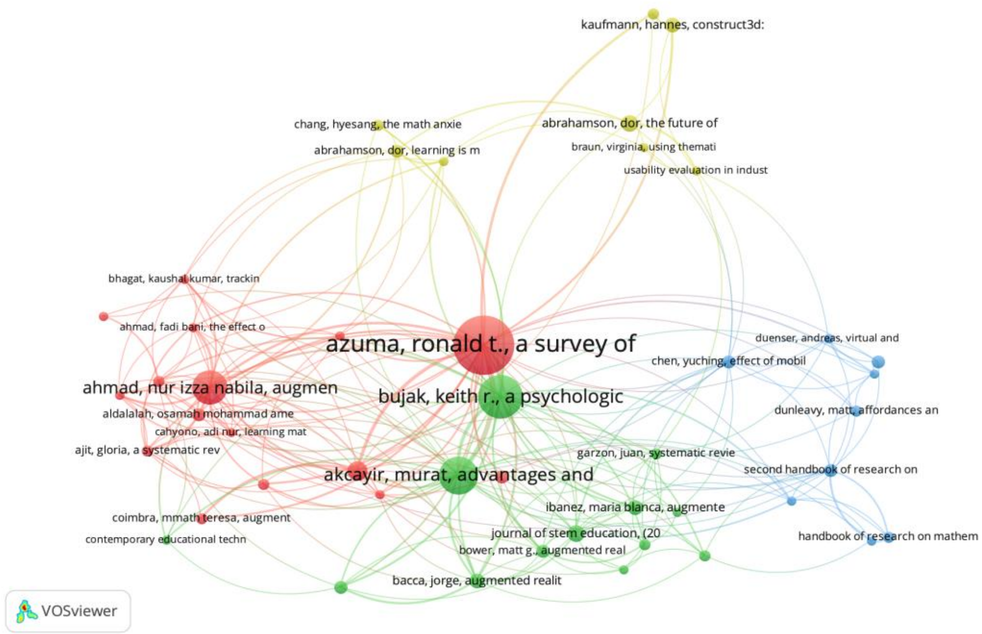

Figure 6: Co-citation Network of Influential Documents

4.9 Co-Citation Clusters and Intellectual Structure of the Field

Figure 6 shows that the co-citation network from VOSviewer has four clear clusters. These clusters highlight the main ideas behind research on augmented reality (AR) and immersive learning technologies in mathematics education. Furthermore, they also show how the field brings together AR theory, educational synthesis, mathematics-specific pedagogical applications, and both embodied and cognitive perspectives.

1. Cluster 1 (Red): Mathematics-Focused AR Applications and Learning Outcomes

The red cluster focuses on empirical and review research examining augmented reality-based mathematics learning outcomes (302) regarding visual thinking, achievement, motivation, and conceptual understanding. Notable examples of this cluster are Bujak et al. (2013); Mahmood et al. (2016) Ahmed and Junaini (2020), Cai et al. (2020), Ahmad (2021) and Arıcan and Özçakır (2020).

Most of these works focus on AR current use in the classroom level, specifically for geometry, proportional reasoning, probability and mathematical modelling. These studies are often cited together and reflect a common thread of interest in how AR can facilitate visualization, spatial reasoning, and emotional factors such as motivation and math anxiety (Bujak et al., 2013; Chen, 2019). In general, this cluster is related to the practical aspect of the field where AR as a tool is used for solving existing problems in learning mathematics.

2. Cluster 2 (Green): Systematic Reviews and Educational Technology Integration

The green cluster is dominated by systematic reviews and integrative studies that synthesize evidence on AR in education and STEM learning. Key references include Bacca et al. (2014), Bower et al. (2014); Akçayır and Akçayır (2017), Ibáñez and Delgado-Kloos (2018), and Garzón and Acevedo (2019).

This cluster represents the core methodological and evaluative literature. Combining research outcomes, listing key benefits and learning opportunities, with principles to follow in defining AR usage. The strong co-citation connections with Cluster 1 suggest sorting AR-related research in mathematics often overlaps and refers to models of educational technology as well as more comprehensive reviews of evidence. Thus, theory-driven and carefully validated immersive learning technologies are growing in popularity in the educational area.

3. Cluster 3 (Blue): Foundational AR Theory and Immersive Learning Frameworks

The blue cluster shows the theoretical and technological underpinnings of augmented reality and immersive learning. Key works include Azuma (1997), Azuma et al. (2001), Dunleavy et al. (2009), Wu et al. (2013), and Billinghurst et al. (2012, 2014).

These studies include AR's defining characteristics, its technological benefits and teaching potential and constitute the conceptual scaffolding necessary for later educational applications. Azuma's centrality suggests that its key works are highly connected to one another and well linked with all groups, linking tech innovation with educational practice. Therefore, this set of studies provides compelling evidence that existing research in mathematics education utilizing AR is largely grounded in fundamental AR theory and principles of immersive learning.

4. Cluster 4 (Yellow): Embodied Cognition, Visualization, and Mathematical Thinking

The yellow cluster highlights research based on the study of embodied cognition, visualization or cognitive activity in learning mathematics. Pioneer works include Arcavi (2003), Clarke (2006), Alibali and Nathan (2012), and Abrahamson (2016; Abrahamson et al., 2020). This cluster reflects an increasing awareness that the learning of mathematics is not merely symbolic, but also embodied, perceptual and action oriented. This aligns with co-citation links between VR and AR-focused clusters indicating that immersive technologies are increasingly being construed through the frame of embodied design and visualization theories, where the learners' physical bodies as well as the senses shape how they come to a mathematical idea (Abrahamson, 2016).

This integration represents a conceptual shift in the design of AR environments, moving beyond content delivery and aligning more closely with embodied mathematical reasoning. This direction of research shows how AR serves as a semiotic bridge that connects the learner's intuitive meanings, or conventional sensorimotor information, to ensure students can progress toward formalized mathematical language through comparing and contrasting one medium with another.

Taken together, the four clusters reveal a coherent and layered intellectual structure. Foundational AR theories (Cluster 3) inform systematic educational syntheses (Cluster 2), which in turn guide mathematics-specific implementations (Cluster 1), increasingly interpreted through embodied and cognitive perspectives (Cluster 4). This progression reflects the maturation of the field from technology-driven exploration toward pedagogically grounded and cognitively informed AR design in mathematics education.

Table 5: Document Co-Citation Clusters and Associated Research Themes

Cluster	Label	No. of Items	Representative Authors / References
Cluster 1 (Red)	**Mathematics-Focused AR Applications and Learning Outcomes**	16	Bujak et al. (2013); Aldalalah et al. (2019); Chen (2019); Ahmad & Junaini (2020); Cahyono et al. (2020); Cai et al. (2020); Ahmad (2021); Arıcan & Özçakır (2020)
Cluster 2 (Green)	**Systematic Reviews and Educational Technology Integration**	13	Billinghurst & Duenser (2012) ; Wu et al. (2013); Bacca et al. (2014) ; Bower et al. (2014); Akçayır & Akçayır (2017); Ibáñez & Delgado-Kloos (2018); Garzón and Acevedo. (2019); Ajit (2021)
Cluster 3 (Blue)	**Foundational Augmented Reality and Immersive Learning Frameworks**	9	Azuma (1997) ; Azuma et al. (2001) ; Billinghurst et al. (2014) ; Dunleavy et al. (2009) ; Wu et al. (2013); Duenser et al. (2006); Kaufmann et al. (2000); Kaufmann (2003)
Cluster 4 (Yellow)	**Embodied Cognition, Visualization, and Mathematical Thinking**	8	Handbook of Research on Mathematics Teaching and Learning (1992); Gittler (1998); Arcavi (2003); Braun & Clarke (2006); Alibali & Nathan (2012); Abrahamson (2016); Abrahamson et al. (2020)

The results of the co-citation analysis demonstrate a coherent intellectual structure on the intersections between AR and immersive learning concerning mathematics education based on four thematic clusters. Cluster 1 (Red) represents the largest area of work focusing on mathematics-specific AR application and learning outcomes, which comprises empirical studies that focus on achievement (N = 7) as well as visual thinking, motivation, and affective variables in AR-supported setting for mathematics learning contexts. Cluster 2 (Green) refers to the consolidation and synthesis phase of the field with systematic reviews and integrative studies exploring AR as well as immersive technologies across educational contexts providing theoretical framing and methodological guidance for future studies.

Cluster 3 (Blue) represents the conceptual underpinnings of more specific applications, recognized to support systems architectures, affordances, pedagogical potentials, and impact or benefit statements that laid down a broader tradition of applied educational research in AR. Lastly, Cluster 4 (Yellow) papers show the theoretical basis behind learning that includes embodied cognition, visualization and mathematical thinking aspects, integrating AR to broader perspectives of learning sciences. All in all, these clusters highlight this evolution of the field from foundational AR technologies and learning theories to systematic synthesis and context-specific applications within mathematics education, pointing out a flourishing research domain that is theoretically grounded.

4.10-Co-word analysis

Co-word analysis is a technique that allows researchers to identify key concepts, their associations, and thematic clusters, providing insights into the knowledge structure and dynamics of a field (Biranvand et al., 2020; Kilic & Uyar, 2022). To examine the semantic structure and thematic focus of the literature, the author keywords were critically analyzed from the bibliometric dataset. From a total of **2,016 unique keywords**, **62 keywords** met the minimum threshold of **nine occurrences**.

Table 6: Top 15 Keywords Identified in the Co-Word Network Analysis

Rank	Keyword	Occurrences	Total Link Strength
1	students	125	832
2	virtual reality	137	766
3	augmented reality	174	765
4	mathematics education	150	748
5	engineering education	76	583
6	teaching	68	505
7	e-learning	75	480
8	science technologies	41	388
9	science education	34	317
10	technology education	29	306
11	engineering and mathematics	29	297
12	learning systems	36	253
13	mathematics learning	53	252
14	education computing	31	251
15	computer-aided instruction	31	226

The co-word analysis reveals a well-defined and conceptually cohesive knowledge structure underlying research on augmented reality (AR) and immersive learning in mathematics education. As shown in Table 6, the most prominent keywords are *students*, *augmented reality*, *virtual reality*, and *mathematics education*, all exhibiting high occurrence frequencies and strong total link strengths. This pattern indicates that the literature is predominantly learner-centered and strongly anchored in the application of immersive technologies to mathematics teaching and learning contexts. The high total link strength of *students* suggests its central role in connecting multiple thematic strands, reflecting a strong emphasis on learning outcomes, engagement, and learner experiences in AR-supported mathematics education.

Technology-oriented terms such as *virtual reality*, *augmented reality*, *e-learning*, and *learning systems* further highlight the field's focus on digital and immersive instructional environments. These strong connections indicate that AR research in mathematics is highly contextualized within wider trends of educational technology and online learning ecologies. Simultaneously, pedagogical and disciplinary keywords, such as like *teaching*, *mathematics learning*, *science education* and *engineering education*, highlight the interdisciplinary nature of the field and connect mathematics with STEM-informed strategies for instruction and design-based curricula.

Moreover, the presence of keywords such as *computer-aided instruction*, *education computing*, and *technology education* indicates a sustained interest in the instructional potentials of technology-enhanced learning tools. Collectively, these results suggest that contemporary research on AR and immersive learning in mathematics education is characterized by a convergence of learner-focused pedagogies, immersive technologies, and STEM integration. The co-word network thus reflects a mature and interconnected research domain in which technological innovation and pedagogical inquiry jointly shape the evolving research agenda.

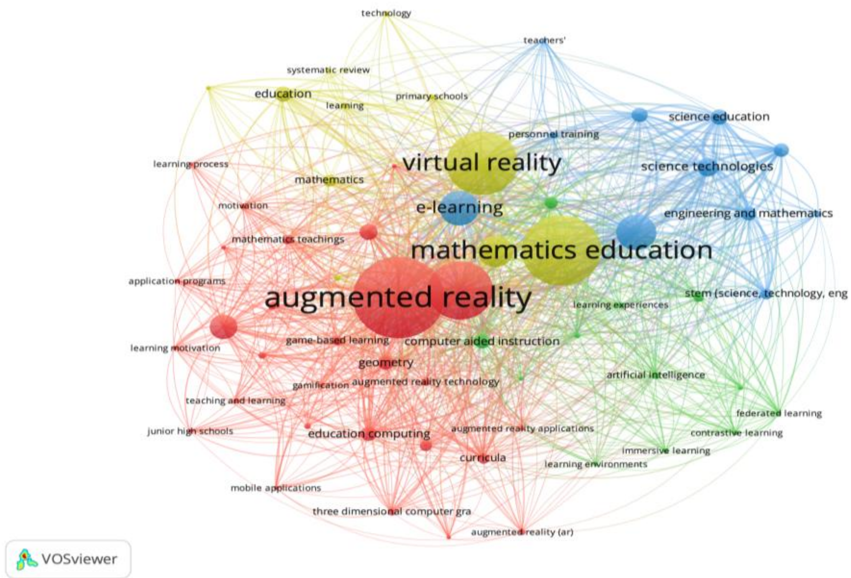

Figure 7: Co-Word Network of Author Keywords

The co-word analysis reveals a structured and thematically coherent knowledge landscape in research on augmented reality (AR) and immersive learning in mathematics education. Using VOSviewer, 62 high-frequency author keywords meeting the minimum threshold of nine occurrences were mapped into a keyword co-occurrence network, resulting in four distinct thematic clusters. These clusters reflect dominant research foci and illustrate how technological, pedagogical, and disciplinary concepts are interlinked across the literature (Biranvand et al., 2020; Kılıç & Uyar, 2022).

1. Cluster 1 (Red): Augmented Reality Applications and Mathematics Learning Processes

The red cluster, centered on the keyword *augmented reality*, represents the largest and most densely connected thematic group. This cluster consists mainly of the keywords like *mathematics learning*, *learning systems*, *computer-aided instruction*, *geometry, curricula*, *mobile applications*, *junior high schools*, and *teaching and learning*. In a broad sense, these terms express the focus of instructional applications of AR in mathematics classrooms, specifically how AR technologies mediate conceptual understanding, visualization and learning processes across education levels. The frequent occurrence of pedagogical and learning-focused keywords indicates that most studies are anchored within practice. They investigate the learning potentials of AR applications for mathematics instruction to increase student engagement.

2. Cluster 2 (Blue): STEM Integration and Technology-Enhanced Science and Mathematics Education

Keywords in the blue cluster include *science education*, *science technologies*, *technology education*, *engineering and mathematics*, *STEM education*, and *teachers*. This cluster demonstrates an interdisciplinary STEM orientation, contextualizing mathematics education with respect to larger issues in science and engineering. The close co-occurrence of math- and science-related terms reflects one application of AR and immersive technologies as integrative tools that transcend traditional disciplinary boundaries. This cluster of research highlights the significance of teacher preparation, technology integration strategies, and scientific pedagogies in support of learning across STEM domains as immersive technologies become a more prominent presence in cross-disciplinary learning environments.

3. Cluster 3 (Green): Immersive, Intelligent, and Advanced Learning Technologies

The green cluster is characterized by keywords associated with advanced digital learning environments, including *immersive learning*, *learning environments*, *learning experiences*, *virtual environments*, *artificial intelligence*, *federated learning*, and *contrastive learning*. This cluster points to an emerging technological trajectory, where AR research intersects with intelligent systems and data-driven learning approaches. The presence of artificial intelligence–related terms suggest a growing interest in augmenting immersive learning with adaptive, personalized, and intelligent features, indicating a forward-looking research agenda that extends beyond traditional AR applications.

The green cluster is composed of keywords connected to advanced digital learning environments, such as *immersive learning*, *learning environments*, *learning experiences*, *virtual environments*, *artificial intelligence*, *federated learning*, and *contrastive learning*. This cluster represents an emerging technological trajectory where AR research meets intelligent systems and data-centric learning paradigms. The presence of artificial intelligence–related terms confirm a trend towards an increasing interest in utilizing immersive learning with adaptive, personalized, and intelligent features foreshadowing a progressive research agenda moving beyond conventional AR applications.

4. Cluster 4 (Yellow): Virtual Reality, E-Learning, and Educational Foundations

Yellow cluster, led by predominant central keyword *virtual reality*, including *e-learning*, *education*, *learning*, *technology*, *primary schools*, and *systematic review*. This cluster forms the basic conceptual and infrastructural components of the field, situating immersive technologies within larger educational technology and online learning frameworks. *Virtual reality* and *e-learning* are engaged as immersive environment for learning, thus often exploring in the field of digital learning and distance education while systematic review denotes synthesis and consolidation of already existing evidence.

To sum up, the four clusters show a complex research environment where AR focused classroom applications (Cluster 1) are embedded within broader STEM and teacher-education contexts (Cluster 2), increasingly intersect with intelligent and immersive learning technologies (Cluster 3), and are grounded in foundational virtual learning and educational technology frameworks (Cluster 4). The strong interconnections among clusters indicate that research on AR and immersive learning in mathematics education has developed into a well-established, interdisciplinary field that integrates pedagogy, technology, and disciplinary knowledge, while also simultaneously moving toward deeper and intelligent learning systems.

Table 7: Co-Word Clusters: Thematic Groupings and Key Publications

Cluster	Cluster Label	No. of Keywords	Representative Keywords
Cluster 1 (Red)	Augmented Reality Applications and Mathematics Learning Processes	28	augmented reality; mathematics learning; learning systems; computer-aided instruction; geometry; curricula; mobile applications; junior high schools; teaching and learning; education computing; visualization
Cluster 2 (Blue)	STEM Integration and Technology-Enhanced Science and Mathematics Education	12	science education; science technologies; technology education; engineering and mathematics; STEM education; teachers; personnel training; engineering education
Cluster 3 (Green)	Immersive, Intelligent, and Advanced Learning Technologies	11	immersive learning; learning environments; learning experiences; virtual environments; artificial intelligence; federated learning; contrastive learning; interactive learning
Cluster 4 (Yellow)	Virtual Reality, E-Learning, and Educational Foundations	11	virtual reality; e-learning; education; learning; technology; primary schools; systematic review; mathematics education

Table 7 summarizes four-co-word clusters that map the thematic structure of the literature on augmented and virtual reality in mathematics education. The clusters reflect (1) the strong emphasis on augmented reality applications and mathematics learning practices, (2) the integration of AR/VR within STEM and science education contexts, (3) the emergence of immersive and intelligent learning environments, and (4) the role of foundational educational concepts, pedagogy, and curriculum design. Collectively, these clusters indicate that the field is shaped by both applied classroom-oriented research and emerging technology-driven innovations that continue to expand the pedagogical possibilities of AR and VR in mathematics education.

4.11 Three-Field Plot Analysis (Keywords–Sources–Countries)

The three-field plot of author keywords (DE), publication sources (SO) and authors' countries (AU_CO) is illustrated in Figure 8, which offers an integrated view of the research themes over key outlets and geographic contexts. The network overview reveals that global terms—especially *augmented reality*, *virtual reality*, *mathematics education*, *educational technology,* and *e-learning*—are used most widely as bridges between journals and countries under investigation in this paper suggesting their foundational position for any further meaningful construction of the field. These themes are primarily tied to technology presenting and education publishing venues, such as conference proceedings and high-impact journals in instructional technology, computer science and STEM education that showcases academic research on immersive technologies for mathematics learning in both pedagogical and technical domains.

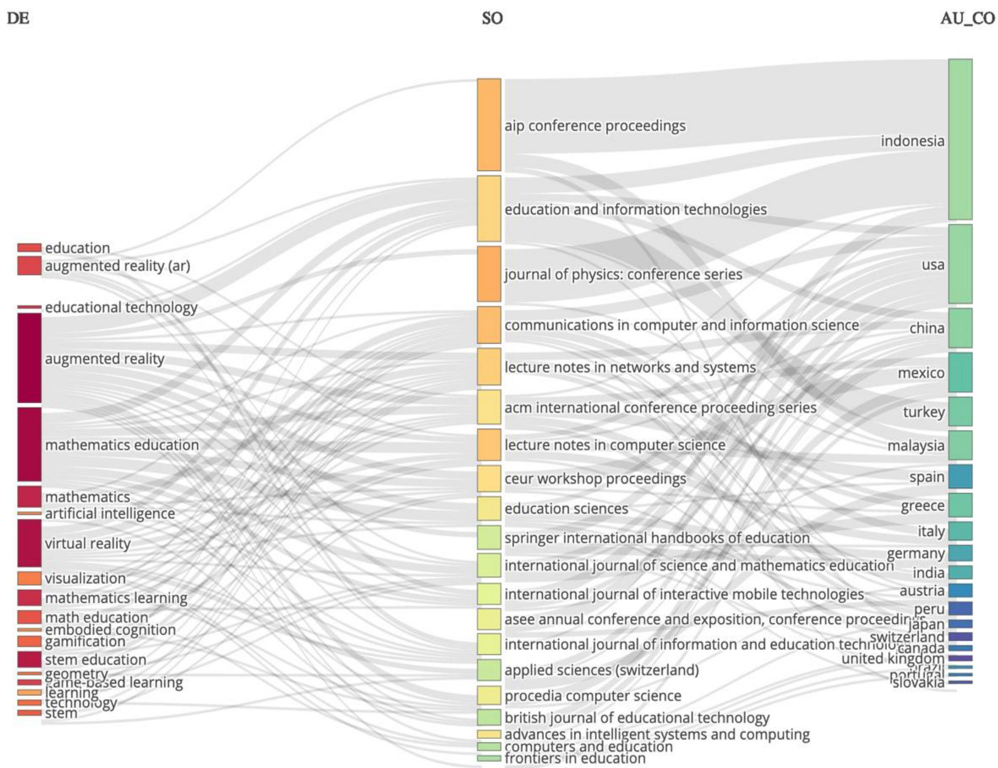

Figure 8: Three-Field Plot of Keywords (DE), Sources (SO), and Author Countries (AU_CO) in Augmented Reality Mathematics Education Research

Geographically, the visualization highlighted the remarkable contributions of Indonesia, the United States, China, and several European countries, utilizing numerous sources and keywords. The structure of the flows reveals a distinct hierarchy in dissemination strategies, while the United States and European nations show a balanced distribution across high-impact journals. This suggests that while Western research output is entrenched in established peer-reviewed periodicals, a significant portion of Asian scholarship, particularly from Indonesia, is driving the rapid expansion of the field through conference proceedings.

This pattern indicates a geographically dispersed but thematically focused research field, in which the same core topics are studied across various national contexts. Emerging new topics, such as AR and mathematics education, become bridging concepts between several publication venues that span a wide range of countries, stating their status as truly global research foci. In a general sense, the three-field plot illustrates the interconnection of conceptual, intellectual, and geographic aspects of literature, thereby promoting the interdisciplinary and international nature of work on immersive technologies in mathematics and STEM education.

Trend Topics

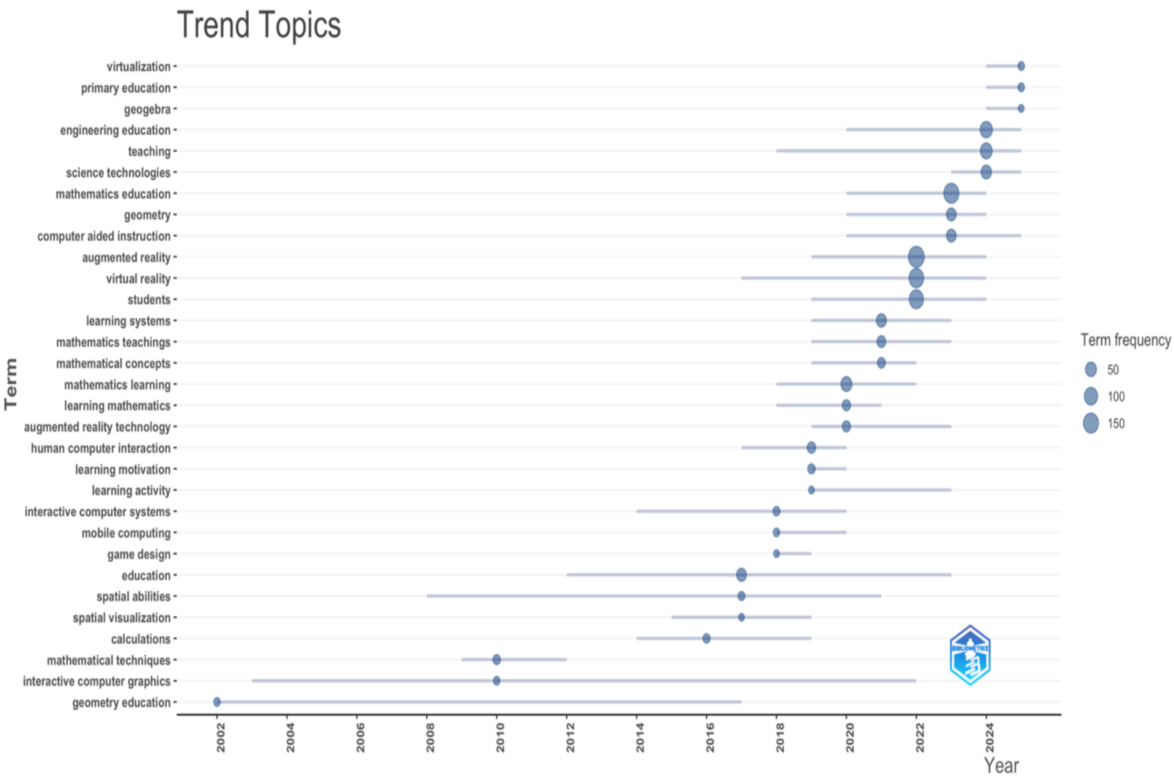

Figure 9: Trend Topics in Augmented and Virtual Reality Research in Mathematics Education (2002–2025)

The trend topics analysis demonstrates a clear temporal development of research themes concerning augmented reality and virtual reality in mathematics and

STEM education. The early research activity (2000–2010) was mainly on basic topics, including geometry education, interactive computer graphics, mathematical techniques, and spatial abilities, evidencing earlier emphasis on visualization, representational tools, and computer-assisted mathematics learning. These earlier themes provided a conceptual foundation for the use of digital technologies in mathematics education.

Since about 2011, the literature exhibits an increase in focus on pedagogical and learner-centered topics, such as learning activity, learning motivation, HCI (human–computer interaction), and computer-aided instruction. This shift reflects an increasing attention towards how learners engage with technology-supported environments and how this engagement affects eLearning and learning processes.

Research activity shows significant increasing trends after 2016, with a noticeable rise of augmented reality, virtual reality, and augmented reality technology as the remaining leading high-frequency topics. These topics show trends for growth well into 2024, agreeing with the maturity of immersive technologies as foundational instruments in mathematics education research. At the same time, highly correlated pedagogical domains (i.e., mathematics education, mathematics learning, teaching and learning systems, and students) become highlighted, indicating a focus from technology development to instructional effectiveness and learning outcomes.

In the latest era, as a wider range of learning contexts (i.e., school education and beyond) are increasingly impacted by immersive technologies, the merging of science education, science, technology/engineering/education, as well as Technology education, is clearly visible. This trend corresponds with a growing incorporation of augmented and virtual reality in interdisciplinary STEM contexts, indicating broad movement toward integrated technological pedagogy rather than isolated interventions.

The trend topics analysis reflects a clear transition from past visualization and computation-directed research to future studies focused on immersive technologies, learner perspective, and STEM integration. The transformation also embodies the move in the field of mathematics toward pedagogically grounded and inter-disciplinary applications of augmented and virtual reality environments for teaching and learning mathematics.

4.12 Overlay Visualization of Keyword Co-Occurrence

Figure 10 presents the overlay visualization of keyword co-occurrence, illustrating the temporal evolution of research themes in augmented and virtual reality–enhanced mathematics and STEM education. In this visualization, node size reflects keyword frequency, link thickness indicates co-occurrence strength, and color gradients represent the average publication year, with cooler tones corresponding to earlier studies and warmer tones indicating more recent research activity.

On the overlay map, prior work, mainly colored with a cooler blue or green scale, focused on basic pedagogical and technological issues such as computer-aided instruction, geometry, mathematics teaching, learning systems, and education computing. These issues represent the beginning of a process in which digital tools are used to enhance visualization, representation, and efficiency of instruction in mathematics education.

As the field developed, the keywords augmented reality, virtual reality, and e-learning have emerged as central nodes and very connected ones. They function as bridges between old instructional technology (Virtual Reality) and newer, more immersive educational technologies. The high frequency and central position of these keywords illustrate their role as the key methodological and pedagogical cornerstones in our research territory.

Recent research trends, indicated by warmer yellow hues, show the focus is increasingly on interdisciplinary and advanced learning contexts such as science education, science technologies, engineering education, engineering and mathematics (i.e. STEM), artificial intelligence systems in education, immersive learning environments or federated learning. This pattern indicates a move toward integration of immersive technologies in a larger context including intelligent learning systems and educational-use at an ecosystem-wide level beyond numbers-driven instruction in mathematics.

The overlay visualization shows an apparent evolving, chronological order in the literature; from early computer and visualization-based pedagogical approaches to more recent themes of immersive, intelligent and interdisciplinary learning environments. The outlined development is much in line with the increased understanding and recognition of AR and VR research, especially in connection to mathematics education. This paper exemplifies new pathways that are directed towards paving bridges between immersive technology together with STEM integration and transformative educational environments.

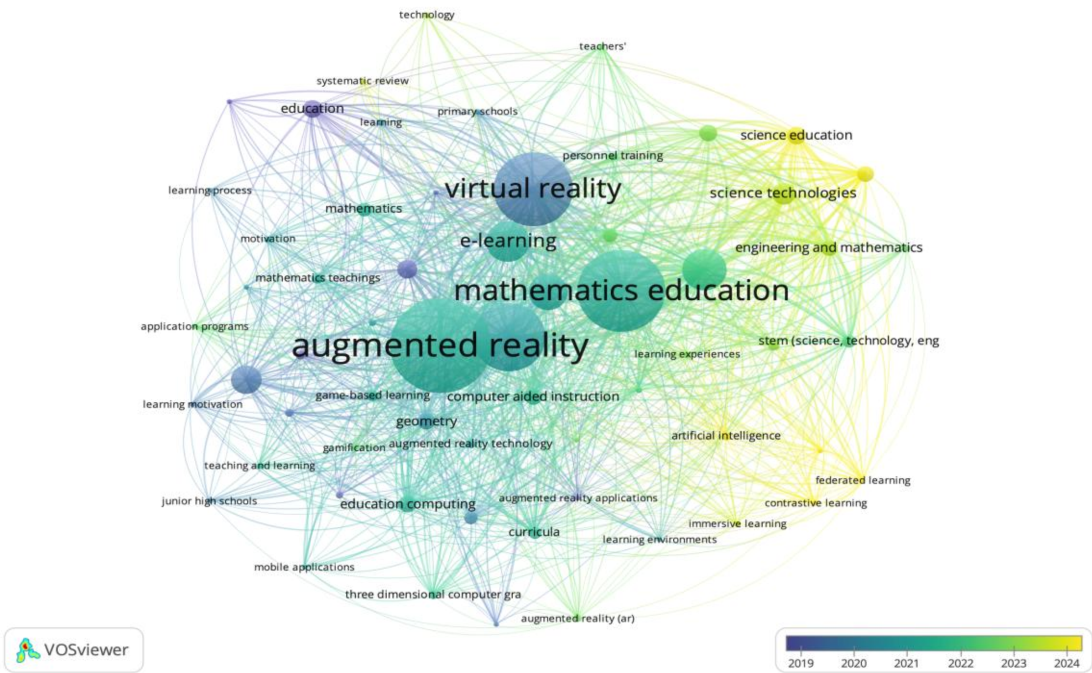

Figure 10: Overlay visualization of keyword co-occurrence in Scopus-indexed articles

5. Discussion

The bibliometric findings overall suggest that AR and immersive technologies in mathematics education research has increasingly become a rapidly expanding and conceptually diversifying field. The continuous increase in scientific production on a yearly basis, especially from 2015 onward, demonstrates the coincidence of technological maturity (i.e. greater availability of immersive tools) and an increasing pedagogical demand for visualization-rich learning environments within STEM education (Bacca et al., 2014; Akçayır & Akçayır, 2017). Comparatively, while Science Education has a long history of virtual inquiry, mathematics education appears to be in a "catch-up" phase, focusing primarily on visualization, rather than the process-skill assessment typical of science virtual laboratories. The relatively young mean age of the documents, as well as this collective work, is not close to theoretical saturation. However, they are more likely still in the process of active formation, with stable experimentation and refinement of pedagogical design.

5.1 Intellectual foundations and consolidation of the field

The citation and co-citation document analyses indicate multilayered patterns of intellectual structure within the literature. Seminal works on AR serve as the conceptual foundation in the discipline, connecting earlier technical understandings of AR with subsequent educational and cognitive applications (Azuma, 1997; Azuma et al., 2001). These references, which are highly co-cited have very high total link strength; thus, they may still exert influence across a variety of research lines. But the high ratio of conference to journal papers suggests that it is still a field determined by rapid prototyping and technical novelty. The prevalence of proceedings suggests that the field has not yet reached methodological rigor or maturity in its disciplines. Most importantly, their continued relevance suggests that contemporary education research is still informed by early insights into AR as a theoretical model, and that it does so even in the context of accelerating technical change over time. These give rise to correctly afford interactions, spatial registration, and real–virtual conjunctions.

Aligned with this groundwork, highly cited narrative reviews and meta-analyses indicate an ongoing effort to establish evidence and unpack the pedagogical implications (Bacca et al., 2014; Akçayır & Akçayır, 2017; Garzón and Acevedo, 2019). Consolidation of this sort is typical in a field that is starting to mature, as research groups graduate from single-individual intervention studies and comparisons among them to design principles and boundary conditions around effectiveness. In complement, the strong showing for math-dedicated AR in the co-citation network (Bujak et al. 2013; Chen, 2019; Cai et al., 2020) underlines a methodological embedding and reinforces a sense that mathematics education has arguably emerged as one of the more pervasively established application spaces over which immersive technologies stake their claim.

5.2 Thematic differentiation: from application to cognition

The structures of the co-citation and co-word clusters suggest that the field is not homogeneous, but rather organized along different, though interrelated thematic trajectories. The dominant cluster is related to mathematics-oriented AR apps and learning achievements, such as achievement, conceptual understanding, visual thinking, and affective components including motivation and anxiety (Chen, 2019; Ahmad & Junaini, 2020; Arıcan & Özçakır, 2020). However, the spatial nature of AR makes geometry king in this domain. Calculus and other areas of mathematics are still underrepresented because it would have been a missed opportunity to visualize dynamics which the literature did not touch upon yet. This emphasis reflects the demand that AR interventions evidence learning gains, and not simply novelty effects.

A second prominent avenue is in AR research of e-learning, learning management systems, and STEM-oriented curricula integrated with wider digital learning environments (Bower et al., 2014; Ibáñez & Delgado-Kloos, 2018). This cluster indicates that immersive technologies seem to be gaining traction as components of integrated instructional systems, instead of isolated supplements. Importantly, many of the recent outputs are Indonesian and as such point to South-East Asia, through a great extent of regional digitization policy impact on global publishing statistics.

The finding of a cluster surrounding embodied cognition, visualization, and mathematical thinking reflects another theoretical turn in the literature. Research informed by embodied and representational perspectives asserts that the pedagogic potential of AR can be explained through its ability to externalize intangible mathematical concepts through body-based interaction, spatial manipulation, and perceptual feedback (Arcavi, 2003; Alibali & Nathan, 2012 Abrahamson, 2016). This theoretical alliance bolsters the explanatory foundation of AR mathematics research, addressing not only "what works" but also how and why immersive experiences engender conceptual change.

5.3 Semantic structure and learner-centered orientation

Co-word analysis also supports these interpretations by offering a semantic core centered on learners. As highlighted in the central network, high-frequency and high-link-strength keywords dominate this map: students, augmented reality, virtual reality, and mathematics education- these keywords illustrate that research attention remains on students' learning experiences as opposed only to technical development. The significant links to teaching, e-learning and learning systems also indicate that AR research is becoming increasingly situated in digitally mediated pedagogical contexts, mirroring the wider shifts occurring within educational praxis (Kılıç & Uyar, 2022).

Data included co-occurrence of terms indicating wide use of different combinations such as "engineering education" to describe the impact on mathematics learning using AR, and suggestions that these types of interventions were often positioned in multidisciplinary or integrated fashion. By calling attention to this point, educators highlight the versatile nature of immersive technologies but also propose that a central contribution of future work will be making clear how mathematics learning is authentic and advanced in interdisciplinary designs, as opposed to being overshadowed by shallow spread of technology.

5.4 Emerging trends and future directions

Continuities and changes in the themes of educational technology was reflected in the most popular topics over time. Evolution is not only noticed in terms of analysis level, but also as a movement from visualization and computer-aided instruction to more advanced themes such as virtual reality and augmented reality, changing focus from visual, even spontaneous, towards student-centered outcomes. This evolution is a mirror of the wider development in this area, from exploratory tools for visualizing data, to immersive, data-driven learning environments that may support more elaborate cognitive and instructional processes. Despite being significant, this level of international co-authorship is not high and indicates that knowledge production remains limited or highly regionalized, thus restricting the possibility for cross-contextual verification of results. At best, without greater input from those under-resourced populations, it risks constructing a gap for privileged perspectives and thus exacerbating the divide between those who mold the future of educational technology and the diverse group that will be excluded from it.

Altogether, the findings sketch out a field that is high-tech but theory literate, n- and innovative but consolidating. Moving the field forward, future research should urgently require theory-driven design studies (Abrahamson, 2016), drawing on models of cognitive and embodied learning, and that also feature longitudinal and scalable implementations that extend up from short interventions with greater international collaboration allowing for deeper experiences of generalizability and methodological rigor. When combining the ICT affordances supporting novelty and engagement in immersive technology, with rich pedagogical frameworks and aligned research agendas, AR can be anti-poverty instruction when reducing to a minimum cost low-investment mathematic education whilst using its innovative youth power find unconventional mathematics learning solutions. Additionally, concrete models like Cognitive Load Theory, TPACK adoption of immersive content knowledge and SAMR Technology Integration framework must be adapted to study how AR artifacts become a mathematical tool.

6. Conclusion

This bibliometric analysis offers a comprehensive overview of the expanding field of research on augmented and immersive technologies in mathematics education. As for publication trends, the data across the three decades shows a solid annual increase of 15.34%, showing that this field has matured from early explorations to theoretical and pedagogically based research. In this case, with respect to the intellectual structure of the field, results show a consolidation of scholarly underpinnings around foundational augmented reality frames but pacing alongside growing focus on student-centered outcomes as well as interdisciplinary convergences and systematic patterns in evidence synthesis.

In particular, the emphasis on visualization, embodied cognition, and cognitive systems positions immersive technologies as a medium that could bolster mathematical understanding by bridging abstract concepts with interactive and experiential modalities of learning. These led into the four thematic clusters including: (1) augmented reality applications and mathematics learning processes, (2) STEM integration and technology-enhanced science and mathematics education, (3) immersive and intelligent learning environments, and (4) virtual reality, e-learning, and educational foundations.

In terms of evolutionary trends, progress in this field will require a transition from standalone intervention studies to scalable, theory-based interventions that are responsive to context and embedded in curriculum and learner populations. Overall, future research needs to further consolidate cognitive and learning theories, develop longitudinal and cross-cultural studies, and promote more international collaboration when applying the findings of one study or context to others. With immersive tech finally attainable, carefully researched integration into mathematics education shows potential for the future of more engaging, equitable, and conceptually rich experiences in the years to come.

7. Recommendations

In light of the bibliometric results and thematic trends revealed in this study, several future research, policy, and practice directions are suggested for augmented/assisted technologies for mathematics education.

First, future research must have a better theoretical underpinning and therefore more explicitly position augmented and immersive learning designs in established learning theories (i.e., embodied cognition, constructivism, metacognition). In so doing, this work has the potential to advance the field beyond instrument-determined experimentation and toward into-theory interpretation, while supporting explications of how and why such technologies afford mathematics learning. To operationalize this, the development of a standardized framework for AR mathematics design is suggested that aligns specific immersive technologies with corresponding mathematical competencies.

In the second instance, researchers are invited to conduct both longitudinal and large-scale empirical studies to explore learning retention, transfer of mathematical understanding, and long-term consequences on learners' motivation as well as spatial reasoning and problem-solving skills. Designs like that would counteract the current short-term-intervention fad and build stronger evidence base for deciding what to do in education. Additionally, researchers should target underrepresented mathematical topics, where AR's ability to visualize complex data distributions, such as in Statistics, could provide unique pedagogical value currently missing in the literature.

Third, more emphasis should be placed on teacher training and development. Preservice and in-service teacher education that supports teachers in designing, implementing, and critically reflecting on augmented and immersive learning activities is vital for pedagogically sound classroom utilization. Studies targeting teachers' pedagogical beliefs, technological skills and access to technology will help ensure more sustainable uptake. Furthermore, policymakers must prioritize necessary VR/AR infrastructure in schools to prevent digital divide.

Fourth, it would be advisable for further studies to broaden their sample and focus across different educational contexts and learner profiles, especially in underprivileged or resource-limited situations. Comparative, cross-cultural research may help in understanding conditions of effectiveness and equity for technology-enhanced mathematics learning. To support this global equity, adopting specific "Open Access" practices to improve global knowledge sharing may be employed.

Lastly, inter-disciplinary cooperation involving mathematics educators, learning science researchers, and technology developers is strongly advised. Such collaborations may enable the creation of learner-centered, curriculum-aligned, and ethically led immersive learning spaces that underpin the use of new technologies to drive innovation but also ensure access and quality for all learners. Crucially, this collaboration must establish ethical guidelines for AI and data-driven immersive environments which safeguards students' privacy. Collectively, these recommendations seek to inform the subsequent wave of research in support of more meaningful, replicable, and theoretically grounded implementations of immersive technologies in mathematics education.

8. Limitations

This study has several limitations that should be considered when interpreting the findings. First, the analysis was restricted to Scopus-indexed publications, which may have excluded relevant studies from other databases and non-indexed regional outputs. Second, bibliometric results depend on the quality and consistency of metadata, and variations in keyword usage or citation practices may have influenced network structures despite normalization efforts. Third, citation-based indicators tend to favor older publications, potentially underrepresenting the influence of recent but emerging studies. Finally, as a bibliometric investigation, this study identifies patterns and trends rather than evaluating the pedagogical quality or effectiveness of individual interventions. These limitations suggest the value of complementary qualitative and empirical research to deepen understanding of immersive technologies in mathematics education.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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